Math 126
Summer 2018
Dr. Lily Yen

Test 3

Show all your work

Name: _____

Number: _____

Signature: _____

Score: ___/40

No Calculator permitted in this part. Read the questions carefully. Show all your work and clearly indicate your final answer. Use proper notation.

Problem 1: Solve for y in the following initial value problem. Express your solution explicitly as $y = f(x)$. Include the domain of f.

$$x^2y' + e^{-y} = 0, \quad y(2) = 4.$$

Score: /5

Problem 2: Find the orthogonal trajectories of the family of curves $kx^5 = y^4$, where k is an arbitrary constant.

Score: /4

Problem 3: Determine whether each sequence below converges as $n \to \infty$. If so, find the limit.

a. $\left\{ \dfrac{3n - (-1)^n}{3n + (-1)^n} \right\}$

b. $a_n = \dfrac{n \sin(n\pi/2)}{2n + 1}$

c. $s_n = \displaystyle\sum_{k=0}^{n} \left(\frac{3^k}{2^{2k}} - \frac{1}{k!} \right)$

d. The sequence a_n satisfying the recurrence: $a_n = 3a_{n-1} - a_{n-2}$ for integers $n \geq 2$ and $a_0 = 1$, and $a_1 = 1$.

Score: /12

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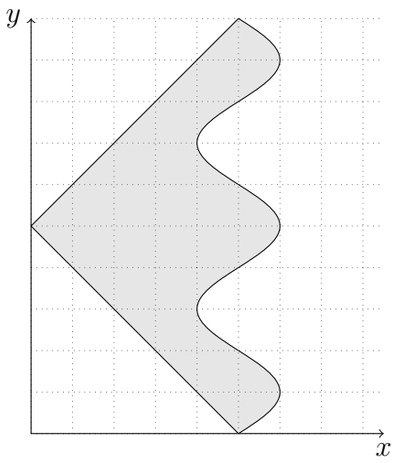
Calculators permitted in this part.

Problem 4: The shaded region shown is bounded between the curves:

$$x = |y - 5|, \qquad \text{and} \qquad x = 5 + \cos(\tfrac{\pi}{2}(y - 5))$$

Use integrals to express the perimeter. Give your answer to 4-decimal place accuracy.



Score: /5

Problem 5: For the series

$$S = \sum_{k=1}^{\infty} \frac{(-1)^k k}{e^k}$$

a. Graph the n-th partial sum of the series for the first 10 values of n.

b. Suppose you use S_{49} to estimate S. Bound the error $|S - S_{49}|$.

c. Determine whether the series S converges absolutely.

Score: /6

Problem 6: Recall Newton's law of cooling: the rate of change in temperature of an object varies proportionally to the difference in temperature between the object and the surrounding medium. Write a differential equation whose solution is the temperature as a function of time for a bottle of frozen elder flower juice taken out of a $-18\,^{\circ}\mathrm{C}$ deep freezer and left in a $20\,^{\circ}\mathrm{C}$ kitchen, given that after one hour, the temperature of the bottle rose $5\,^{\circ}\mathrm{C}$. Include all conditions. Solve the equation for the temperature of the object at time t and graph the solution.

Score: /8