Math 126
Summer 2018
Dr. Lily Yen

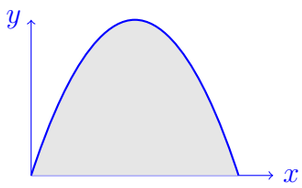
Test 1

Show all your work

Name: _____
Number: _____
Signature: _____
Score: ___/43

No Calculator permitted in this part. Read the questions carefully. Show all your work and clearly indicate your final answer. Use proper notation.

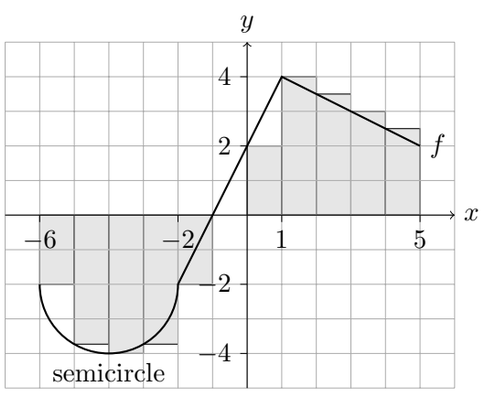
Problem 1: Find the expression of a Riemann sum with n intervals using MIDPOINT rectangular approximation method to estimate the area bounded by $f(x) = 3x - x^2$ and the x-axis. Draw the region of interest.



$$\lim_{n\to\infty} \frac{3}{n}\sum_{j=1}^{n} f\left(\frac{3}{n}\left(j-\frac{1}{2}\right)\right) = \lim_{n\to\infty} \frac{3}{n}\sum_{j=1}^{n} \frac{9}{n}\left(j-\frac{1}{2}\right) - \frac{9}{n^2}\left(j-\frac{1}{2}\right)^2$$

Score: /4

Problem 2: Evaluate the following integrals exactly.



a. $\displaystyle\int_0^5 f(x)\,dx$

A triangle over a rectangle:
$\frac{1}{2}\cdot 5\cdot 2 + 5\cdot 2 = 5 + 10 = 15.$

b. $\displaystyle\int_{-6}^0 f(x)\,dx$

The two triangle on the right cancel each other out, so you are left with a semicircle below a rectangle:
$-\left(\frac{1}{2}\pi 2^2 + 4\cdot 2\right) = -2\pi - 8.$

c. $\displaystyle\int_{-6}^5 |f(x)|\,dx$

This time the two triangles do not cancel, so you have the answer to parts a and b above plus the two triangles:
$(15) + (2\pi + 8) + 2(\frac{1}{2}\cdot 1\cdot 2) = 25 + 2\pi.$

d. $\displaystyle\left|\int_{-6}^5 f(x)\,dx\right|$

From parts a and b,
$\int_{-6}^5 f(x)\,dx = (-2\pi - 8) + (15) = 7 - 2\pi,$
which is positive, so
$\left|\int_{-6}^5 f(x)\,dx\right| = 7 - 2\pi.$

e. Draw L_{11} directly on the grid.

Score: /7

Problem 3: Find $g(t)$ in

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{\sqrt{2+\frac{5k}{n}}}{n} = \int_2^7 g(t)\,dt$$

This is the right-hand Riemann sum for

$$g(t) = \tfrac{1}{5}\sqrt{t}$$

Score: /2

Problem 4: Integrate the following. Use correct notation and show all steps.

a. $\displaystyle\int \frac{dt}{3t+5}$

If $u = 3t+5$, then $\frac{du}{dt} = 3$, so $dt = \frac{1}{3}\,du$. Therefore

$$\int \frac{1}{3t+5}\,dt = \int \frac{1}{u}\,\frac{1}{3}\,du = \tfrac{1}{3}\int \frac{1}{u}\,du + C = \tfrac{1}{3}\ln|u| + C = \tfrac{1}{3}\ln|3t+5| + C$$

Score: /2

b. $\displaystyle\int \sqrt[4]{x}(4x^2 - 3x^3)\,dx$

$$\int \sqrt[4]{x}(4x^2-3x^3)\,dx = \int x^{1/4}(4x^2-3x^3)\,dx = \int 4x^{9/4}-3x^{13/4}\,dx = \tfrac{16}{13}x^{13/4}-\tfrac{12}{17}x^{17/4}+C$$

Score: /3

c. $\displaystyle\int \sin(\theta)\cos(\theta)e^{\cos^2(\theta)+1}\,d\theta$

If $u = \cos^2(\theta)+1$, then $\frac{du}{d\theta} = 2\cos(\theta)(-\sin(\theta)) = -2\sin(\theta)\cos(\theta)$, so $d\theta = -\frac{du}{2\sin(\theta)\cos(\theta)}$. Therefore

$$\begin{aligned}\int \sin(\theta)\cos(\theta)e^{\cos^2(\theta)+1}\,d\theta &= \int \sin(\theta)\cos(\theta)e^{u}\frac{-1}{2\sin(\theta)\cos(\theta)}\,du \\ &= -\tfrac{1}{2}\int e^u\,du = -\tfrac{1}{2}e^u + C = -\tfrac{1}{2}e^{\cos^2(\theta)+1} + C\end{aligned}$$

Score: /3

d. $\displaystyle\int_1^e \frac{\ln(x)}{x}\,dx$

If $u = \ln(x)$, then $\frac{du}{dx} = \frac{1}{x}$, so $dx = x\,du$. Therefore

$$\int_1^e \frac{\ln(x)}{x}\,dx = \int_{x=1}^{x=e}\frac{u}{x}x\,du = \int_{u=0}^{u=1} u\,du = \tfrac{1}{2}u^2\big|_0^1 = \frac{1}{2}$$

Score: /3

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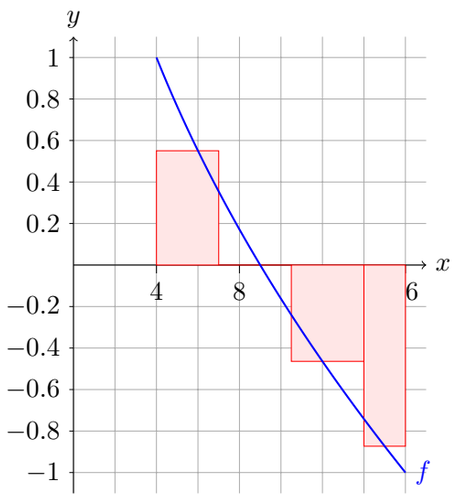
A TI83 or TI84 Calculator permitted in this part. Read the questions carefully. Show all your work and clearly indicate your final answer. Use proper notation.

Problem 5: Given the function $g(x) = 3 - \sqrt{x}$, the partition $P = \{4, 7, 10.5, 14, 16\}$, and its corresponding sample points $C = \{6, 9, 12, 15\}$, compute its Riemann sum. You need to draw the graph and its approximating rectangles, then write out each term in the sum before using the calculator to find the answer.



$(7-4)g(6)+(10.5-7)g(9)+(14-10.5)g(12)+(16-14)g(15) = 3(3-\sqrt{6})+3.5(0)+3.5(3-\sqrt{12})+2(3-\sqrt{15}) \approx -1.719$

Score: /3

Problem 6: One day a rainstorm came to Vancouver pouring at a rate of $r(t) = 17+2t-t^2$ in mm/h, where t is the number of hours since midnight.

a. Estimate when the rain stopped.

Solving $r(t) = 0$ yields $t = 5.24$ or $t = -3.24$. Since t is the number of hours since midnight, only the positive solution is meaningful. Thus the rain stopped around 5:15 am.

b. Use the table to estimate the total rainfall.

Approx	$n = 10$	$n = 100$	$n = 200$
L_n	72.79	69.02	68.80
R_n	63.88	68.13	68.35

Total 68.58 mm

Score: /4

Problem 7: An astronomer estimates that in a certain constellation, the number of stars per magnitude m, per degree-squared of the sky, is equal to $A(m) = 2.400 \times 10^{-6} m^{7.4}$. (Fainter stars have higher magnitudes.) Determine the total number of stars of magnitude between 6 (just visible to the naked eye) and 15 (much too faint to be seen with binoculars) in a 1 degree-squared region of the sky.

$$\int_6^{15} 2.400 \times 10^{-6} m^{7.4}\, dm = \frac{2.400 \times 10^{-6}}{8.4} m^{8.4}\Big|_6^{15} = 2162$$

Score: /3

Problem 8: Use the Fundamental Theorem of Calculus to find the following.

a. If $G(x) = \displaystyle\int_0^{x^3} \sqrt{t+1}\,dt$, find $G'(x)$.

Let $h(x) = \int_0^{x^3} \sqrt{t+1}\,dt$. Then $h'(x) = \sqrt{x+1}$ by the FTC. Moreover, $G(x) = h(x^3)$, so $G'(x) = h'(x^3)3x^2 = 3x^2\sqrt{x^3+1}$.

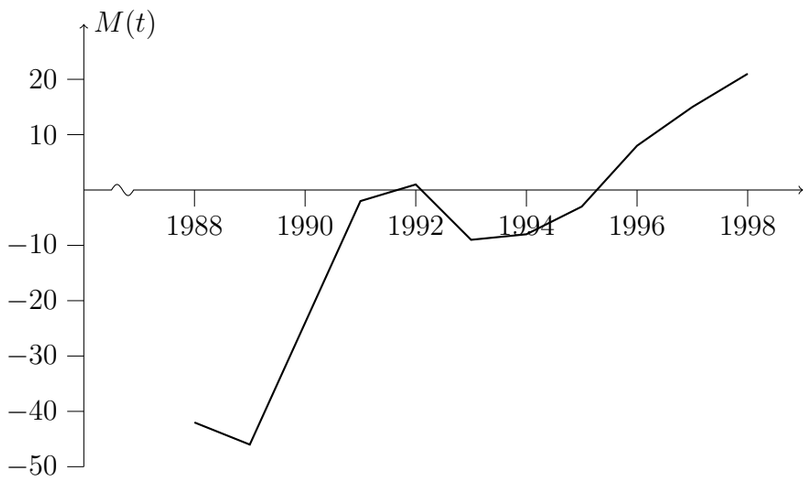
Score: /2

b. If $H(x) = \displaystyle\int_{4x^2}^{9} \frac{1}{t}\,dt$, find $H'(1)$.

Let $g(x) = \int_9^x \frac{1}{t}\,dt$. Then $g'(x) = \frac{1}{x}$ by the FTC. Moreover, $-g(x) = -\int_9^x \frac{1}{t}\,dt = \int_x^9 \frac{1}{t}\,dt$, so $H(x) = -g(4x^2)$. Therefore $H'(x) = -g'(4x^2)8x = -8x\frac{1}{4x^2} = -\frac{2}{x}$. Hence $H'(1) = -2$.

Score: /3

Problem 9: The figure show the migration rate $M(t)$ of Ireland in the period 1988–1998. This is the rate at which people (in thousands per year) move into ($M(t) > 0$) or out of ($M(t) < 0$) the country.



a. Is the integral $\int_{1988}^{1998} M(t)\,dt$ positive or negative? What does this quantity represent?

The integral is negative and represents the net number of people who moved out of Ireland in that decade.

b. Did migration in the period 1988–1998 result in a net influx of people into Ireland or a net outflow of people from Ireland?

Outflow.

c. During which year(s) could the Irish prime minister announce, "We've hit an inflection point; We are still losing population, but the trend is now improving"?

For the migration to hit an inflection point, the rate ($M(t)$) must be at an extremum. For the trend to be improving, that extremum must be a minimum. This happened in 1989 and 1993.

Score: /4