Math 126
Summer 2016
Dr. Lily Yen

Test 1

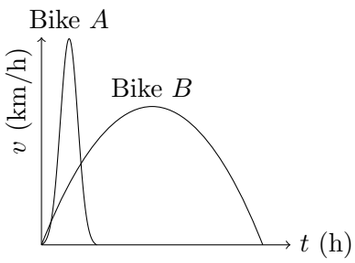
Show all your work

Name: _____

Score: ___/50

No Calculator permitted in this part. Read the questions carefully. Show all your work and clearly indicate your final answer. Use proper notation.

Problem 1: Two bicycles start at the same time and travel in the same direction along the Central Valley Greenway. The graph below gives the velocity, v, of each bicycle as a function of time, t.



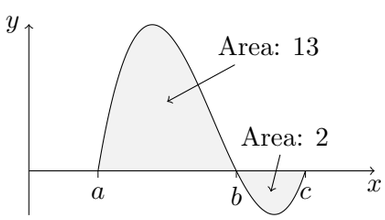
a. Which bicycle attains the larger maximum velocity? A

b. Which bicycle covers the most distance? B

c. Which bicycle reaches the higher acceleration? A

Score: /3

Problem 2: Using the function f graphed here, find the value of each of the following.



a. $\displaystyle\int_b^c f(x)\,dx =$ -2

b. $\displaystyle\int_b^a f(x)\,dx =$ -13

c. $\displaystyle\int_a^c f(x)\,dx =$ 11

d. $\displaystyle\int_a^c |f(x)|\,dx =$ 15

Score: /4

Problem 3: Express the following limit as a definite integral. Describe what your definite integral represents in terms of area.

$$\lim_{n\to\infty}\left(\sum_{k=1}^{n}\ln\left(1+\frac{(e-2)k}{n}\right)\frac{e-2}{n}\right)$$

This is the right-hand Riemann sum for

$$\int_1^{e-1}\ln(x)\,dx$$

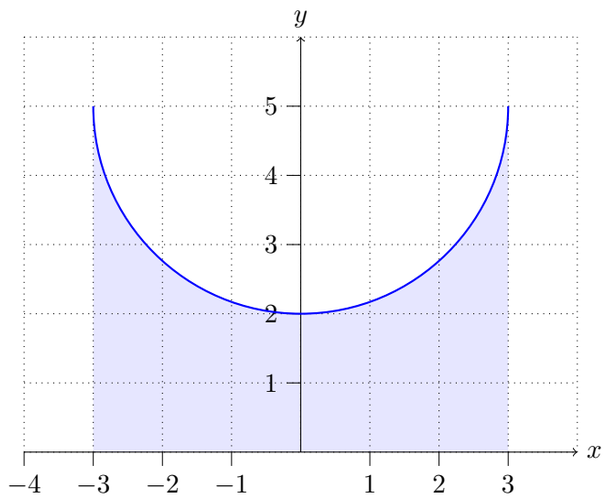
Score: /3

so the area under $y=\ln(x)$ between $x=1$ and $x=e-1$.

Problem 4: Use geometric properties to evaluate the following integral exactly. Please draw the shape on the grid provided.

$$\int_{-3}^{3} (5 - \sqrt{9 - x^2})\, dx$$

So the region is a 6×5 rectangle with a semicircle removed. Thus the area is $30 - \frac{9}{2}\pi \approx 15.9$.



Score: /4

Problem 5: Evaluate the following integrals exactly using the Fundamental Theorem of Calculus.

a. Define $A(x) = \int_{x/3}^{5} (u^2 - e^u)\, du$.

(1) Evaluate $A(3)$.

$A(3) = \int_1^5 (u^2 - e^u)\, du = \frac{1}{3}u^3 - e^u \Big|_1^5 = (\frac{125}{3}) - (e^1 - \frac{1}{3}) = \frac{124}{3} - e^5 + e \approx -104$

(2) Find $A'(x)|_{x=0}$.

Let $h(x) = \int_5^x (u^2 - e^u)\, du$. Then $A(x) = -h(\frac{1}{3}x)$ and by the FTC, $h'(x) = x^2 - e^x$. Therefore $A'(x) = -h'(\frac{1}{3}x) \cdot \frac{1}{3} = -\frac{1}{3}\big((\frac{x}{3})^2 - e^{x/3}\big) = -\frac{x^2}{27} + \frac{1}{3}e^{x/3}$, so $A'(0) = \frac{1}{3}$.

(3) Find $\frac{d}{dx}A(5)$.

$A(5)$ is a constant, so its derivative is 0.

Score: /5

b. Given $g(x) = \int_{3-\cos x}^{2x^3} \frac{1+t}{1-t}\, dt$, find $g'(x)$.

As in Part a2 above, let $h(x) = \int_0^x \frac{1+t}{1-t}\, dt$. Then $h'(x) = \frac{1+x}{1-x}$ and $g(x) = h(2x^3) - h(3 - \cos(x))$, so $g'(x) = h'(2x^3)6x^2 - h'(3 - \cos(x))\sin(x) = 6x^2 \frac{1+2x^3}{1-2x^3} - \frac{4-\cos(x)}{\cos(x)-2}\sin(x)$.

Score: /2

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Calculators permitted from here on.

Problem 6: Integrate the following analytically.

a. $\displaystyle\int \frac{\sin(t)}{\cos^4(t)}\,dt$

If $u = \cos(t)$, then $\frac{du}{dt} = -\sin(t)$, so $dt = -\frac{1}{\sin(t)}\,du$. Therefore
$\int \frac{\sin(t)}{\cos^4(t)}\,dt = \int \frac{\sin(t)}{u^4}\frac{-1}{\sin(t)}\,du = \int \frac{-1}{u^4}\,du = \frac{1}{3u^3} + C = \frac{1}{3\cos^3(t)} + C$

Score: /2

b. $\displaystyle\int x(2-5x)^{3/4}\,dx$

If $u = 2-5x$, then $x = \frac{1}{5}(2-u)$, so $\frac{dx}{du} = -\frac{1}{5}$ and $dx = -\frac{1}{5}\,du$. Therefore
$\int x(2-5x)^{3/4}\,dx = \int \frac{1}{5}(2-u)u^{3/4}\frac{-1}{5}\,du = \frac{-1}{25}\int 2u^{3/4} - u^{7/4}\,du =$
$\frac{-1}{25}(\frac{8}{7}u^{7/4} - \frac{4}{11}u^{11/4}) + C = -\frac{8}{175}(2-5x)^{7/4} + \frac{4}{275}(2-5x)^{11/4} + C.$

Score: /3

c. $\displaystyle\int_1^6 \frac{1+2x}{\sqrt{x+3}}\,dx$

If $u = x+3$, then $x = u-3$ and $dx = du$. Therefore
$\int_1^6 \frac{1+2x}{\sqrt{x+3}}\,dx = \int_{x=1}^{x=6} \frac{1+2(u-3)}{\sqrt{u}}\,du = \int_4^9 \frac{2u-5}{\sqrt{u}}\,du = \int_4^9 2u^{1/2} - 5u^{-1/2}\,du = \frac{4}{3}u^{3/2} - 10u^{1/2}\Big|_4^9 =$
$(\frac{4}{3}\cdot 27 - 10\cdot 3) - (\frac{4}{3}\cdot 8 - 10\cdot 2) = 36 - 30 - \frac{32}{3} + 20 = 26 - \frac{32}{3} = 15\frac{1}{3} = \frac{46}{3}.$

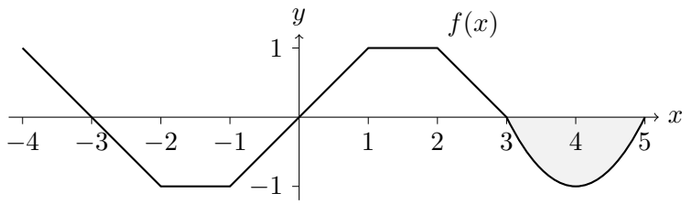
Score: /3

d. $\displaystyle\int \frac{t}{1+25t^4}\,dt$

If $u = 5t^2$, then $\frac{du}{dt} = 10t$, so $dt = \frac{1}{10t}\,du$. Therefore $\int \frac{t}{1+25t^4}\,dt = \int \frac{t}{1+(5t^2)^2}\,dt =$
$\int \frac{t}{1+u^2}\cdot\frac{1}{10t}\,du = \frac{1}{10}\int \frac{1}{1+u^2}\,du = \frac{1}{10}\tan^{-1}(u) + C = \frac{1}{10}\tan^{-1}(5t^2) + C.$

Score: /3

Problem 7: Refer to this figure:



a. Find $\int_{-3}^{0} f(x)\,dx =$ $\boxed{-2}$.

b. If the area of the shaded region is A, estimate $\int_{-3}^{4} f(x)\,dx =$ $\boxed{-\frac{1}{2}A}$.

c. Define $G(x) = \int_{-4}^{x} f(t)\,dt$. Answer the following for G.

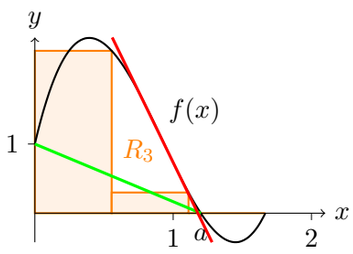
(1) $G(-1) =$ $\boxed{-1}$.

(2) $G'(\frac{1}{2}) =$ $\boxed{\frac{1}{2}}$.

(3) For which x-value(s) does G attain its maximum? $x =$ $\boxed{-3, 3}$.

Score: /5

Problem 8: Use the graph below to order the following from least to greatest.



a. $f'(1)$

b. The average value of $f(x)$ on $0 \le x \le a$.

c. The average value of the rate of change of $f(x)$, for $0 \le x \le a$.

d. $\int_0^a f(x)\,dx$

The tangent at $x = 1$ is much steeper than the average rate of change. Thus $a < c < 0$. On the other hand, $\int_0^a f(x)\,dx > 0$, and since $a > 1$, the average value $\frac{1}{a-0}\int_0^a f(x)\,dx$ is smaller than $\int_0^a f(x)\,dx$. Thus $0 < b < d$.
Hence

$$a < c < b < d$$

Score: /3

Directly on the graph provided, draw in and shade the approximating rectangles representing R_3.

Score: /1

Problem 9: In May in a small plot, Morning Glories and Buttercups germinate at the rate of $1.032e^{0.378t}$ per day where $t = 1$ measures the germination of such weeds at the end of day 1, namely, May 1st.

a. When weeding was not done daily, how many plants would germinate during the first week in May? Express your answer as an integral and evaluate to the nearest integer.

$\int_0^7 1.032e^{0.378t}\,dt \approx 35.8 \approx 36$

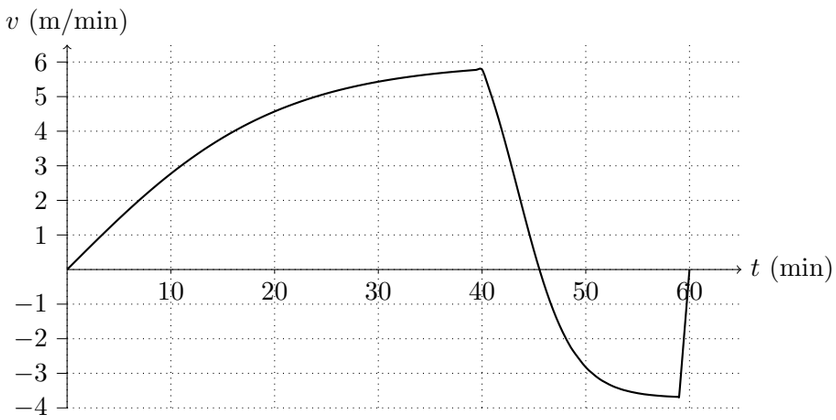
Score: /2

b. How many germinated on Mother's Day (May 8th)? State how you obtain the number to the nearest integer.

$\int_7^8 1.032e^{0.378t}\,dt \approx 17.7 \approx 18.$
If you instead estimate using the rate at, say $t = 7.5$, you get
$1.032e^{0.378\times 7.5} = 17.6 \approx 18.$

Score: /1

Problem 10: The Montgolfier brothers (Joseph and Étienne) were eighteenth-century pioneers in the field of hot-air ballooning. Had they had the appropriate instruments, they might have left us a record, like that shown below, of one of their early experiments. The graph shows their vertical velocity, v, with upward as positive.



a. Give interval(s) where acceleration was positive.

The velocity was increasing on $(0, 40) \cup (59, 60)$

b. Find the time when the greatest altitude was achieved. What was the altitude?

The altitude was maximal when $v = 0$, so after 45.5 min. The greatest altitude was the area under the curve up till that time, so about 176 m.

c. When was the upward acceleration greatest?

The graph of v is steepest during the last minute.

d. What might have happened during this flight to explain when deceleration was greatest? The Montgolfier brothers cut the tethers and their balloon rises swiftly. The rate slows likely because their heating apparatus isn't quite up to the task of keeping the hot-air balloon hot. Then, 40 min into their flight, a catastrophic leak causes the balloon to (first slow, then) careen toward the ground. The remnants of the balloon act as a parachute slowing the descent until the gondola crashes into the treetops and falls to the ground in the last minute.

e. This particular flight ended on top of a hill. How do you known that it did, and what was the height of the hill above the starting point?

The area below the curve over $(0, 45.5)$ is much larger than the area above the curve under $(45.5, 60)$. In fact, $\int_0^{60} v\,dt \approx 135$ m.

Score: /6