Math 126
Summer 2016
Dr. Lily Yen

Test 1

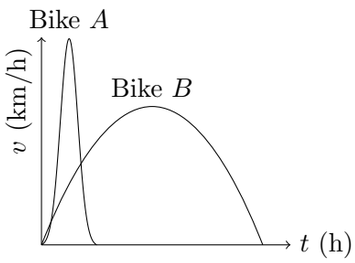
Show all your work

Name: _____

Score: ___/50

No Calculator permitted in this part. Read the questions carefully. Show all your work and clearly indicate your final answer. Use proper notation.

Problem 1: Two bicycles start at the same time and travel in the same direction along the Central Valley Greenway. The graph below gives the velocity, v, of each bicycle as a function of time, t.



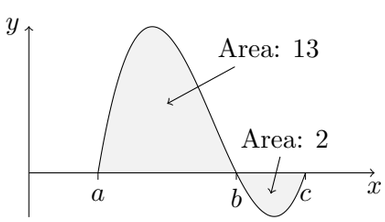
a. Which bicycle attains the larger maximum velocity? ☐

b. Which bicycle covers the most distance? ☐

c. Which bicycle reaches the higher acceleration? ☐

Score: /3

Problem 2: Using the function f graphed here, find the value of each of the following.



a. $\displaystyle\int_b^c f(x)\,dx =$ ☐

b. $\displaystyle\int_b^a f(x)\,dx =$ ☐

c. $\displaystyle\int_a^c f(x)\,dx =$ ☐

d. $\displaystyle\int_a^c |f(x)|\,dx =$ ☐

Score: /4

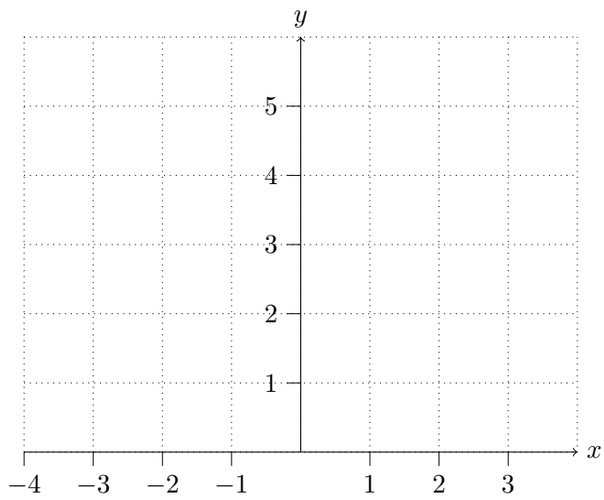
Problem 3: Express the following limit as a definite integral. Describe what your definite integral represents in terms of area.

$$\lim_{n\to\infty}\left(\sum_{k=1}^{n}\ln\left(1+\frac{(e-2)k}{n}\right)\frac{e-2}{n}\right)$$

Score: /3

Problem 4: Use geometric properties to evaluate the following integral exactly. Please draw the shape on the grid provided.

$$\int_{-3}^{3} (5 - \sqrt{9 - x^2})\, dx$$



Score: /4

Problem 5: Evaluate the following integrals exactly using the Fundamental Theorem of Calculus.

a. Define $A(x) = \int_{x/3}^{5} (u^2 - e^u)\, du$.

(1) Evaluate $A(3)$.

(2) Find $A'(x)|_{x=0}$.

(3) Find $\frac{d}{dx}A(5)$.

Score: /5

b. Given $g(x) = \int_{3-\cos x}^{2x^3} \frac{1+t}{1-t}\, dt$, find $g'(x)$.

Score: /2

Math 126
Summer 2016
Dr. Lily Yen

Test 1

Show all your work

Name: _____

Calculators permitted from here on.

Problem 6: Integrate the following analytically.

a. $\displaystyle\int \frac{\sin(t)}{\cos^4(t)}\,dt$

Score: /2

b. $\displaystyle\int x(2-5x)^{3/4}\,dx$

Score: /3

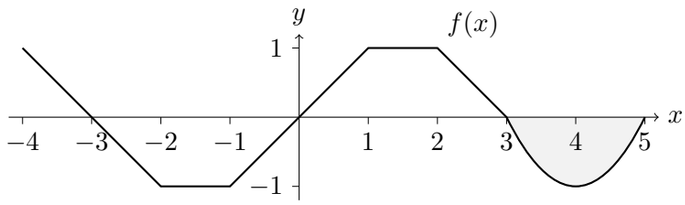
c. $\displaystyle\int_1^6 \frac{1+2x}{\sqrt{x+3}}\,dx$

Score: /3

d. $\displaystyle\int \frac{t}{1+25t^4}\,dt$

Score: /3

Problem 7: Refer to this figure:



a. Find $\int_{-3}^{0} f(x)\,dx =$ [ ].

b. If the area of the shaded region is A, estimate $\int_{-3}^{4} f(x)\,dx =$ [ ].

c. Define $G(x) = \int_{-4}^{x} f(t)\,dt$. Answer the following for G.

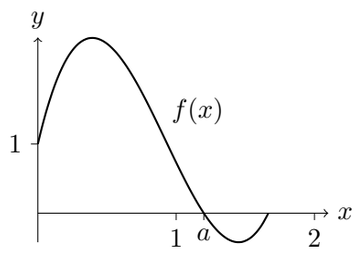
(1) $G(-1) =$ [ ].

(2) $G'(\frac{1}{2}) =$ [ ].

(3) For which x-value does G attain its maximum? $x =$ [ ].

Score: /5

Problem 8: Use the graph below to order the following from least to greatest.



a. $f'(1)$

b. The average value of $f(x)$ on $0 \le x \le a$.

c. The average value of the rate of change of $f(x)$, for $0 \le x \le a$.

d. $\int_0^a f(x)\,dx$

Score: /3

Directly on the graph provided, draw in and shade the approximating rectangles representing R_3.

Score: /1

Problem 9: In May in a small plot, Morning Glories and Buttercups germinate at the rate of $1.032e^{0.378t}$ per day where $t = 1$ measures the germination of such weeds at the end of day 1, namely, May 1st.

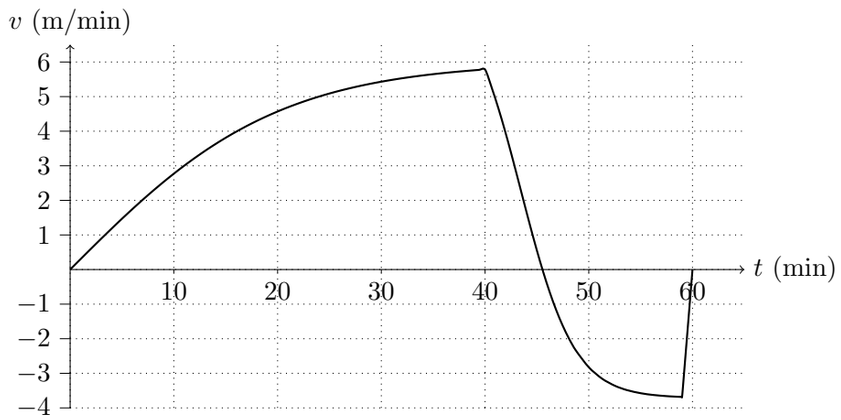
a. When weeding was not done daily, how many plants would germinate during the first week in May? Express your answer as an integral and evaluate to the nearest integer.

Score: /2

b. How many germinated on Mother's Day (May 8th)? State how you obtain the number to the nearest integer.

Score: /1

Problem 10: The Montgolfier brothers (Joseph and Étienne) were eighteenth-century pioneers in the field of hot-air ballooning. Had they had the appropriate instruments, they might have left us a record, like that shown below, of one of their early experiments. The graph shows their vertical velocity, v, with upward as positive.



a. Give interval(s) where acceleration was positive.

b. Find the time when the greatest altitude was achieved. What was the altitude?

c. When was the upward acceleration greatest?

d. What might have happened during this flight to explain when deceleration was greatest?

e. This particular flight ended on top of a hill. How do you known that it did, and what was the height of the hill above the starting point?

Score: /6