Math 126
Summer 2015
Dr. Lily Yen

Test 3

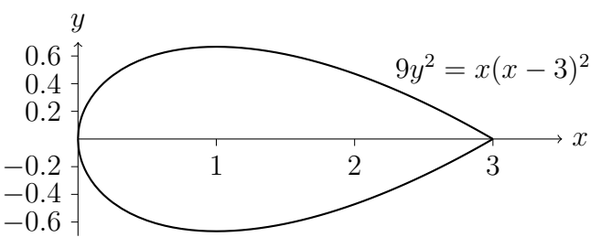
Show all your work

Name: _____

Score: ___/44

No Calculator permitted in this part. Read the questions carefully. Show all your work and clearly indicate your final answer. Use proper notation.

Problem 1: Find the exact arc length of the curve shown.



Score: /5

Problem 2: Solve the following initial value problem.

$$(1-t)\frac{dy}{dt} - y = 0, \qquad y(2) = -4.$$

Score: /4

Problem 3: State the logistic differential equation and discuss how it differs from the exponential differential equation with respect to modelling population growth.

Score: /4

Problem 4: For each sequence or series below, determine if it converges; find the limit if possible, or prove that it diverges.

a. $a_n = \dfrac{(-1)^n n^3 + 2^{-n}}{4^{-n} + 5n^3}$

Score: /3

b. $a_0 = 0, a_1 = 1, a_n = a_{n-1} + a_{n-2}$ for all integers $n \geq 2$.

Score: /3

c. $\displaystyle\sum_{n=0}^{\infty} \frac{2^n - 8}{3^n}$

Score: /4

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Calculators permitted in this part.

Problem 5: Find the orthogonal trajectories of the families of curves where k is a parameter. Sketch at least three members of each family on the same coordinate system. Label each curve with its parameter value.

$$3x - y^3 = k$$

Score: /6

Problem 6: Determine the sum of the series: $\sum_{n=1}^{\infty} \ln(1 + \frac{1}{n})$.

Score: /3

Problem 7: Determine whether the series converges absolutely, conditionally, or not at all. Draw the first 5 partial sums on the coordinate plane for each problem.

a. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$

Score: /3

b. $\displaystyle\sum_{n=2}^{\infty} \frac{\cos(n\pi)}{(\ln(n))^2}$

Score: /3

Problem 8: A 100 L tank contains 50 L of a salt-water solution, with salt concentration of 0.03 kg/L of water. At time zero, pure water is poured into the tank at a rate of 6 L/min and simultaneously, the thoroughly mixed solution is drained from the tank at a rate of 10 L/min. How long does it take to have half the initial amount of salt left in the tank? Provide your answer to the nearest minute.

Score: /6