Math 126
Summer 2015
Dr. Lily Yen

Test 1

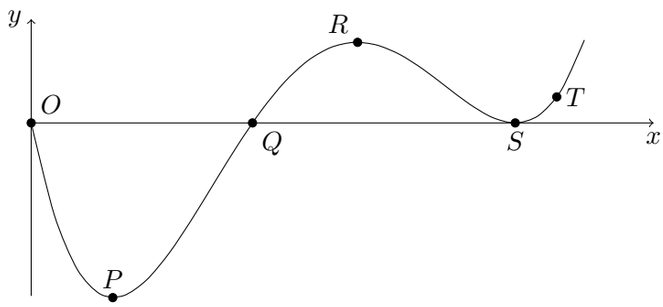
Show all your work

Name: _____

Score: ___/50

No Calculator permitted in this part. Read the questions carefully. Show all your work and clearly indicate your final answer. Use proper notation.

Problem 1: Let $A(x) = \int_0^x f(t)\,dt$, where f is the function graphed below (and x is in the interval shown).



a. $A(x)$ has a local maximum at the x-coordinates of (circle all that apply):

$$O \qquad P \qquad Q \qquad R \qquad S \qquad T$$

b. $A(x)$ has a local minimum at the x-coordinates of (circle all that apply):

$$O \qquad P \qquad Q \qquad R \qquad S \qquad T$$

c. $A(x)$ has points of inflection at the x-coordinates of (circle all that apply):

$$O \qquad P \qquad Q \qquad R \qquad S \qquad T$$

d. Which inequality is true for all x in the interval over which the graph of f is shown? (Circle all that apply.)

$$A(x) \le 0 \qquad A(x) \ge 0 \qquad \text{neither} \qquad \text{both}$$

e. $A(x)$ has an absolute minimum at the x-coordinates of (circle all that apply):

$$O \qquad P \qquad Q \qquad R \qquad S \qquad T$$

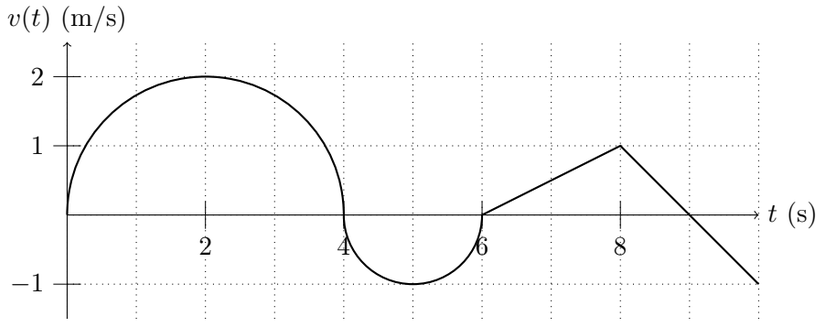
Score: /5

Problem 2: Describe the area represented by the following limit as an integral. Do not evaluate.

$$\lim_{N\to\infty} \frac{\pi}{2N} \sum_{j=1}^{N} \cos\left(\frac{\pi}{3} - \frac{\pi}{4N} + \frac{j\pi}{2N}\right)$$

Score: /3

Problem 3: A particle begins at the origin at time $t = 0$ and moves with velocity $v(t)$ as shown below. (The graph consists of semicircles and line segments.)



a. Find the distance travelled by the particle after 6 seconds.

b. Find the displacement of the particle after 8 seconds.

c. Find the time the particle is farthest from the origin.

d. Directly on the graph, draw in and shade the approximating rectangles representing M_5.

Score: /5

Problem 4: Evaluate the following integrals exactly using the Fundamental Theorem of Calculus.

a. $\dfrac{d}{dt}\displaystyle\int_{100}^{t} \sec(5x - 9)\,dx$

b. $\displaystyle\int \frac{d}{dx}(23x^2 - 8e^x + \cos(x^2))\,dx$

c. $\dfrac{d}{du}\displaystyle\int_{-u}^{\sqrt{u}} \tan\Big(\frac{1}{1+x^2}\Big)\,dx$

Score: /4

Problem 5: Evaluate exactly $\int_1^3 |2x - 4|\,dx$. Sketch the graph.

Score: /2

Problem 6: Find an expression of a function f and a corresponding interval $[a, b]$ such that

$$\left| \int_a^b f(x)\,dx \right| = \frac{1}{2} \quad \text{and} \quad \int_a^b |f(x)|\,dx = \frac{3}{2}.$$

Score: /3

Problem 7: Integrate the following analytically.

a. $\displaystyle\int \frac{2x^2e^x - 3\sqrt{x} + 6}{x^2}\,dx$

Score: /3

b. $\displaystyle\int (x+1)(x^2+2x)^{3/4}\,dx$

Score: /2

c. $\displaystyle\int \frac{\tan(\ln(x))}{x}\,dx$

Score: /3

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Calculators permitted from here on.

Problem 8: Express the following integral as a limit of sums.

$$\int_1^4 e^x \sin(x)\, dx$$

Score: /3

Problem 9: Graph $x = 3y - y^2$ and find the exact area bounded by the curve and y-axis.

Score: /4

Problem 10: Evaluate the following integral exactly.

$$\int_{-3}^{10} \frac{1}{4 + 25x^2}\, dx$$

Score: /4

Problem 11: The rate (in litres per minute) at which water drains from a tank is recorded at half-minute intervals. Compute the average left- and right-endpoint approximations to estimate the total amount of water drained during the first 3 minutes.

t (min)	0	0.5	1	1.5	2	2.5	3
r (L/min)	50	48	46	44	42	40	38

Score: /5

Problem 12: Wind engineers have found that wind speed v (in metres per second) at a given location follows a Rayleigh distribution of the type

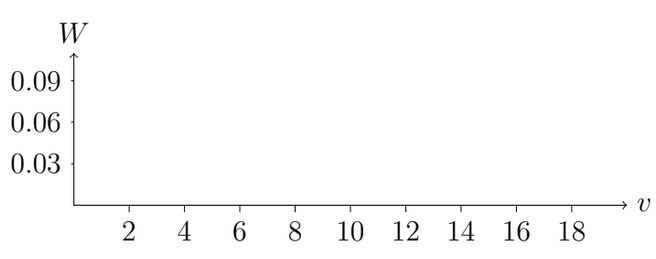
$$W(v) = \frac{1}{32} v e^{-v^2/64}.$$

This means that at a given moment in time, the probability that v lies between a and b is equal to the shaded area under W in the interval $[a, b]$.

a. Find analytically the probability that $v \in [0, b]$.

Score: /3

b. Compute the probability that $v \in [1, 3]$ to 6 decimal place accuracy. Show the shaded region on the graph of W.



Score: /1