Math 126
Summer 2007
Dr. Lily Yen

Test 1

Show all your work

Name: _____

Score: ___/53

No Calculator permitted in this part. Read the questions carefully. Show all your work and clearly indicate your final answer. Use proper notation.

Problem 1: Integrate the following indefinite integrals analytically.

a. $\int \frac{5x}{\sqrt[3]{1-2x}} dx$ Score: /3

$$\frac{3}{4}(1-2x)^{5/3} - \frac{15}{8}(1-2x)^{2/3} + c$$

b. $\int \frac{\cos(x)}{1+\sin^2(x)}\, dx$ Score: /3

$$\arctan(\sin(x)) + c$$

c. $\int t^2 e^t\, dt$ Score: /3

$$t^2e^t - 2te^t + 2e^t + c$$

d. $\int \sin(x)\cos(\cos(x))\, dx$ Score: /3

$$-\sin(\cos(x)) + c$$

e. $\displaystyle\int \frac{3x-1}{x^2-x-6}\,dx$ Score: /3

$$\frac{7}{5}\ln|x+2| + \frac{8}{5}\ln|x-3| + c$$

Problem 2: Evaluate the following definite integrals exactly using the Fundamental Theorem of Calculus.

a. $\displaystyle\int_0^1 x^3(2-3x^4)^4\,dx$ Score: /3

11/20

b. $\displaystyle\int_e^{e^6} \frac{dx}{x\sqrt{\ln x}}$ Score: /3

$2\sqrt{6}-2$

c. $\displaystyle\int_{\pi^2/4}^{\pi^2} \sin(\sqrt{x})\,dx$ Score: /3

$2\pi - 2$

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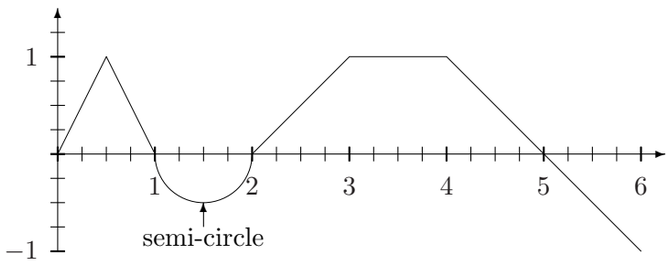
Calculators permitted from here on.

Problem 3: Use the graph of the function f to evaluate the following exactly.



a. $\displaystyle\int_0^2 f(x)\,dx = \frac{1}{2} - \frac{\pi}{8} \approx 0.1073$

b. $\displaystyle\int_2^6 f(x)\,dx = \frac{3}{2}$

c. $\displaystyle\int_0^6 |f(x)|\,dx = 3 + \frac{\pi}{8} \approx 3.3927$

d. $\displaystyle\left|\int_0^6 f(x)\,dx\right| = \left|2 - \frac{\pi}{8}\right| = 2 - \frac{\pi}{8} \approx 1.6073$

e. $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} f\left(1 + \frac{i}{n}\right)\frac{1}{n} = \int_1^2 f(x)\,dx = -\frac{\pi}{8} \approx -0.3927$

f. Calculate L_4 for f on $[2, 6]$. Write out the sum in terms of specific function evaluations (i.e., $f(x_k)$) to show which x_k's you use for each interval before showing the actual function values and final sum.

$L_4 = (f(2) + f(3) + f(4) + f(5)) \cdot 1 = 0 + 1 + 1 + 0 = 2.$

g. Suppose that f gives the velocity in metres per second of a particle as it travels along a straight line. Assuming time is measured in seconds, what is the displacement of the particle between $t = 0$ and $t = 6$.

The displacement is $\int_0^6 f(t)\,dt = 2 - \frac{\pi}{8} \approx 1.6073$ metres.

Score: /10

Problem 4: Show $\int_0^\infty e^{-x^2}\,dx$ is convergent. Score: /3

Since $e^{x^2} > x^2$ for all x, you have that $e^{-x^2} < 1/x^2$. Since $\int_1^\infty \frac{1}{x^2}\,dx = \frac{-1}{x}\Big|_1^\infty = 1 < \infty$, it follows by comparison that $\int_1^\infty e^{-x^2}\,dx$ converges. Clearly $\int_0^1 e^{-x^2}\,dx$ exists, so $\int_0^\infty e^{-x^2}\,dx$ exists.

Problem 5: Estimate $\int_0^{10} f(x)g'(x)\,dx$ if $f(x) = x^2$ and g has the values in the following table. Specify which approximation you use.

x	0	2	4	6	8	10
$g(x)$	2.3	3.1	4.1	5.5	5.9	6.1

Score: /3

By integration by parts,
$\int_0^{10} f(x)g'(x)\,dx = f(x)g(x)|_0^{10} - \int_0^{10} f'(x)g(x)\,dx = x^2g(x)|_0^{10} - \int_0^{10} 2xg(x)\,dx$. Now
$x^2g(x)|_0^{10} = 100 \cdot 6.1 - 0 = 610$, and using the left Riemann sum,
$\int_0^{10} 2xg(x)\,dx \approx \sum_{j=0}^{4} 4j\, g(2j) \cdot 2 = (0 + 4 \cdot 3.1 + 8 \cdot 4.1 + 12 \cdot 5.5 + 16 \cdot 5.9) \cdot 2 = 411.2$.
(This is an underestimate since $2xg(x)$ is increasing.) Thus
$\int_0^{10} f(x)g'(x)\,dx \approx 610 - 411.2 = 198.8$.

Problem 6: If f is continuous and $\int_0^9 f(x)\,dx = 4$, find $\int_0^3 xf(x^2)\,dx$. Score: /3

Let $u = x^2$. Then $\frac{du}{dx} = 2x$, and by substitution,
$\int_0^3 xf(x^2)\,dx = \int_{x=0}^{x=3} xf(u)\frac{1}{2x}\,du = \int_0^9 \frac{1}{2}f(u)\,du = \frac{1}{2} \cdot 4 = 2$.

Problem 7: State both parts of the Fundamental Theorem of Calculus.

Theorem *Let f be a continuous function in $[a, b]$, then*

a. $\dfrac{d}{dx}\displaystyle\int_a^x f(t)\,dt = f(x)$.

b. $\displaystyle\int_a^b f(x)\,dx = F(b) - F(a)$, *where F is an anti-derivative of f, $F'(x) = f(x)$.*

Score: /2

Problem 8: Evaluate exactly:

a. $\displaystyle\int \frac{d}{dx}\left(\sin^6\left(\frac{1}{\sqrt{x}}\right)\right)\,dx$ Score: /1

$$\sin^6\left(\frac{1}{\sqrt{x}}\right)+c$$

b. $\displaystyle\frac{d}{dx}\int_{2x}^{7} t^7-2\sqrt[3]{t}+\csc(t)\,dt$ Score: /1

$$-2\left((2x)^7-2\sqrt[3]{2x}+\csc(2x)\right)$$

c. $\displaystyle\frac{d}{du}\int_{e^u}^{\sin(u)}\frac{x+\ln x}{\cos(x^2)}\,dx$ Score: /2

$$\cos(u)\frac{\sin(u)+\ln(\sin(u))}{\cos(\sin^2(u))}-e^u\frac{e^u+u}{\cos(e^{2u})}$$

d. $\displaystyle\frac{d}{dx}\int_{-\pi}^{e} 3u^2+9\,du$ Score: /1

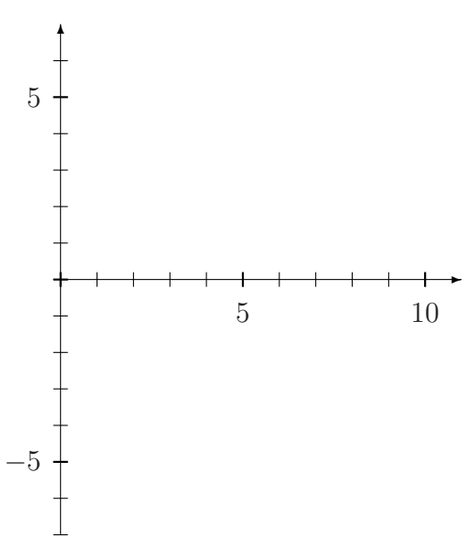
$$0$$

Problem 9: An integral function F is given by $F(x)=\int_0^x f(t)\,dt$. Draw a possible graph of the function $y=f(t)$ (with f continuous) on the interval $[0,10]$ so that F has the following properties:

a. F is decreasing on $[3,8]$, and increasing on both $[0,3]$ and $[8,10]$.

b. $F'(5)=-4$.

c. F is concave down on $[1,4]$, otherwise concave up.



Score: /3