Math 126
Summer 2007
Dr. Lily Yen

Test 1

Show all your work

Name: _____

Score: ___/53

No Calculator permitted in this part. Read the questions carefully. Show all your work and clearly indicate your final answer. Use proper notation.

Problem 1: Integrate the following indefinite integrals analytically.

a. $\int \frac{5x}{\sqrt[3]{1-2x}} dx$ Score: /3

b. $\int \frac{\cos(x)}{1+\sin^2(x)}\, dx$ Score: /3

c. $\int t^2 e^t\, dt$ Score: /3

d. $\int \sin(x)\cos(\cos(x))\, dx$ Score: /3

e. $\displaystyle\int \frac{3x-1}{x^2-x-6}\,dx$ Score: /3

Problem 2: Evaluate the following definite integrals exactly using the Fundamental Theorem of Calculus.

a. $\displaystyle\int_0^1 x^3(2-3x^4)^4\,dx$ Score: /3

b. $\displaystyle\int_e^{e^6} \frac{dx}{x\sqrt{\ln x}}$ Score: /3

c. $\displaystyle\int_{\pi^2/4}^{\pi^2} \sin(\sqrt{x})\,dx$ Score: /3

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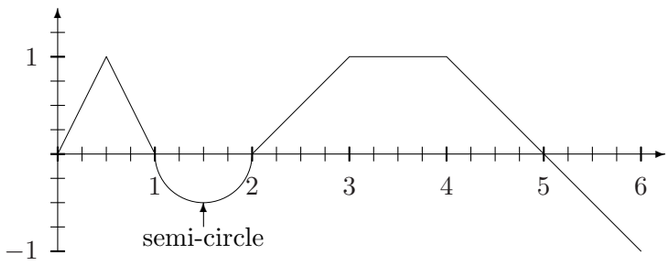
Calculators permitted from here on.

Problem 3: Use the graph of the function f to evaluate the following exactly.



a. $\displaystyle\int_0^2 f(x)\,dx =$

b. $\displaystyle\int_2^6 f(x)\,dx =$

c. $\displaystyle\int_0^6 |f(x)|\,dx =$

d. $\displaystyle\left|\int_0^6 f(x)\,dx\right| =$

e. $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n} f\left(1+\frac{i}{n}\right)\frac{1}{n} =$

f. Calculate L_4 for f on $[2,6]$. Write out the sum in terms of specific function evaluations (i.e., $f(x_k)$) to show which x_k's you use for each interval before showing the actual function values and final sum.

g. Suppose that f gives the velocity in metres per second of a particle as it travels along a straight line. Assuming time is measured in seconds, what is the displacement of the particle between $t = 0$ and $t = 6$.

Score: /10

Problem 4: Show $\int_0^\infty e^{-x^2}\,dx$ is convergent. Score: /3

Problem 5: Estimate $\int_0^{10} f(x)g'(x)\,dx$ if $f(x) = x^2$ and g has the values in the following table. Specify which approximation you use.

x	0	2	4	6	8	10
$g(x)$	2.3	3.1	4.1	5.5	5.9	6.1

Score: /3

Problem 6: If f is continuous and $\int_0^9 f(x)\,dx = 4$, find $\int_0^3 xf(x^2)\,dx$. Score: /3

Problem 7: State both parts of the Fundamental Theorem of Calculus.

Theorem *Let f be a continuous function in $[a, b]$, then*

a.

b.

Score: /2

Problem 8: Evaluate exactly:

a. $\displaystyle\int \frac{d}{dx}\left(\sin^6\left(\frac{1}{\sqrt{x}}\right)\right)\,dx$ Score: /1

b. $\displaystyle\frac{d}{dx}\int_{2x}^{7} t^7 - 2\sqrt[3]{t} + \csc(t)\,dt$ Score: /1

c. $\displaystyle\frac{d}{du}\int_{e^u}^{\sin(u)} \frac{x+\ln x}{\cos(x^2)}\,dx$ Score: /2

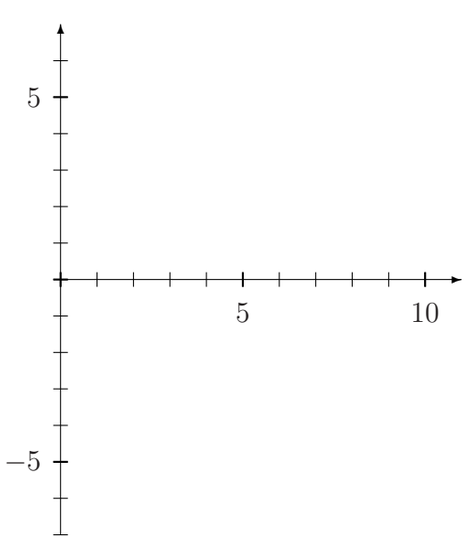
d. $\displaystyle\frac{d}{dx}\int_{-\pi}^{e} 3u^2 + 9\,du$ Score: /1

Problem 9: An integral function F is given by $F(x) = \int_0^x f(t)\,dt$. Draw a possible graph of the function $y = f(t)$ (with f continuous) on the interval $[0, 10]$ so that F has the following properties:

a. F is decreasing on $[3, 8]$, and increasing on both $[0, 3]$ and $[8, 10]$.

b. $F'(5) = -4$.

c. F is concave down on $[1, 4]$, otherwise concave up.



Score: /3