Math 126
Fall 2025
Dr. Lily Yen

Test 3

Show all your work

Full Name: _____
Student Number: _____
Signature: _____
Score: ___/40

No Calculator permitted in this part. Read the questions carefully. Show all your work and clearly indicate your final answer. Use proper notation.

Problem 1: Determine a_3, a_4, and a_5 of the sequence starting with $n = 1$.

$$a_1 = 1, \quad a_2 = 2, \quad a_n = 3a_{n-1} - 2a_{n-2}.$$

Score: /3

Problem 2: Solve the following initial-value problem.

$$t\frac{dy}{dt} = \sqrt{1 - y^2}, \quad (t, y) = (e, 0)\,.$$

Score: /3

Problem 3: Determine the limit of the sequence or show that it diverges.

a. $G_n = \dfrac{3}{5} - \dfrac{9}{25} + \dfrac{27}{125} - \dfrac{81}{625} + \cdots + (-1)^{n-1}\dfrac{3^n}{5^n}$, for integers $n \geq 1$.
For partial marks, write G_1 and G_2 without computing.

Score: /4

/10

b. $S_n = 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}}$.

Score: /3

Problem 4: Determine whether the series converges conditionally, converges absolutely, or diverges.

a. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n\sqrt{n}}$. Write out a few terms before starting your analysis.

Score: /4

b. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{5^n}$. Hint: Consecutive term ratio.

Score: /3

/10

Test 3

Show all your work

Full Name: _____

Student Number: _____

Calculators permitted in this part.

Problem 5: Find the orthogonal trajectories of the families of curves where k is a parameter. Sketch at least three members of each family (the original and the orthogonal) on the same coordinate system. Label each curve with its corresponding parameter (k) value.

$$y = \frac{1}{x+k}$$

Score: /5

Problem 6: Find the sum of the series below by first graphing the nth partial sum for the first 10 n's. State your function entry and window dimensions in your graphing calculator.

$$\frac{2}{1\times 3} + \frac{2}{3\times 5} + \frac{2}{5\times 7} + \frac{2}{7\times 9} + \cdots$$

The denominator of each term in the summand is the product of consecutive odd integers.

Score: /5

/10

Problem 7: Jasmine bought a cup of Tim Horton's coffee which started with a temperature of 75 °C. She let it cool for 10 minutes and measured the temperature to be 70 °C. Suppose the surrounding temperature was 20 °C, find the temperature of her coffee an hour after her purchase.

Score: /4

Problem 8: A pool holds 5000 litres of pure water. A tap is opened and a solution containing 0.2 g of chlorine per litre is fed into the pool for disinfection at a rate of 2 litres per minute. At the same time, a drain plug in the bottom of the pool is opened and the thoroughly mixed solution leaves at the same rate. How much chlorine is in the pool after two hours?

Score: /6

/10