Math 126
Fall 2022
Dr. Lily Yen

Test 2

Show all your work

Name: _____
Number: _____
Signature: _____
Score: ___/35

No Calculator permitted in this part. Read the questions carefully. Show all your work and clearly indicate your final answer. Use proper notation.

Problem 1: Evaluate the following integrals analytically.

a. $\int \tan^{-1} x \, dx$

Score: /2

b. $\int e^x \sin(3x) \, dx$

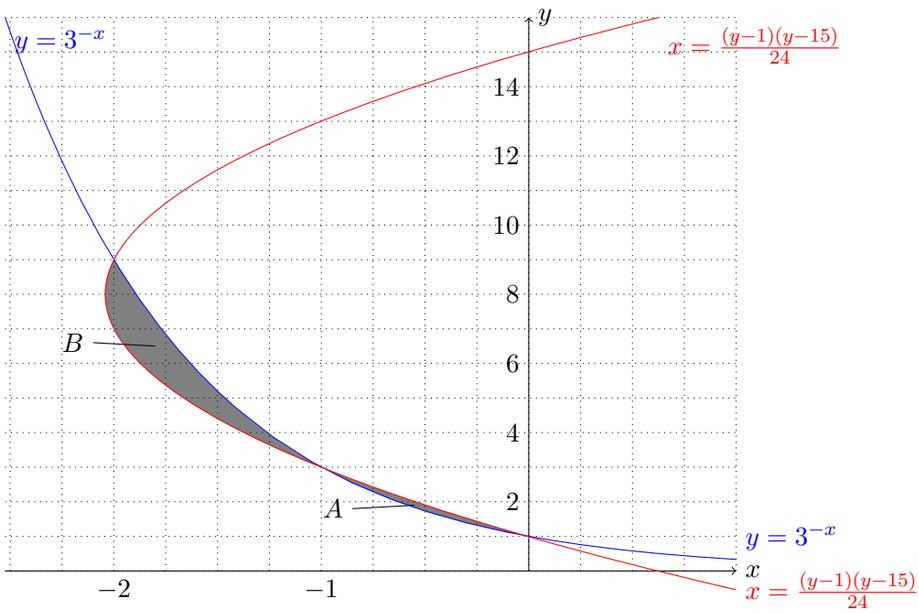
Score: /4

Problem 2: Evaluate if possible; otherwise, state why the integral does not exist.

$$\int_2^4 \frac{1}{(x-3)^2} \, dx$$

Score: /3

Problem 3: Shown are the graphs of $y = 3^{-x}$ and $x = \dfrac{(y-1)(y-15)}{24}$ on the grid including all three intersections for these two curves: $(0, 1)$, $(-1, 3)$, and $(-2, 9)$. The two regions enclosed by the curves are A and B.



Use integrals to express the following. DO NOT EVALUATE YOUR INTEGRALS. Draw a cross-sectional strip for each solid of rotation.

a. The total area of the shaded regions A and B.

Score: /2

b. The volume of a solid that has the shaded region B as its base, and cross-sections perpendicular to the y-axis are equilateral triangles.

Score: /2

c. The volume of the solid obtained by rotating the region A around $x = -2$. State the method used.

Score: /2

d. The volume of the solid obtained by rotating the region B around $y = 10$. State the method used.

Score: /2

Math 126
Fall 2022
Dr. Lily Yen

Test 2

Show all your work

Name: _____

Number: _____

Calculators permitted from here on.

Problem 4: A hanging cable has the shape of $y = \dfrac{e^x + e^{-x}}{2}$ for x in the interval $[-4, 4]$. Draw the curve and set up the integral that expresses the length of the cable. DO NOT EVALUATE YOUR INTEGRAL.

Score: /3

Problem 5: A variable force of $f(x) = \sqrt{3 + x}$ lb is applied in moving an object from $x = 6$ to $x = 13$ feet. How much work is performed? Include units.

Score: /3

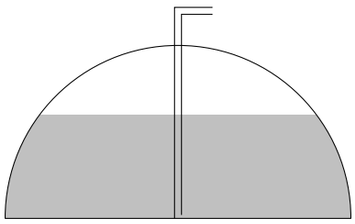
Problem 6: Find the integral analytically: $\displaystyle\int \frac{x^2 + 4x - 1}{(x - 3)(x^2 + 2)}\, dx$

Score: /5

Problem 7: A 20 m uniform chain with a mass density of 1.5 kg/m is dangling from the roof of a building. How much work is required to pull a quarter (1/4) of the chain up onto the top of the building? Use $g = 9.81\,\text{m/s}^2$. Provide your answer accurate to 2 decimal places.

Score: /3

Problem 8: A hemispherical dome with radius 5 m is filled with water to a depth of 3 m. A spout rises 2 m from the top at the centre. Express as a definite integral the work required to pump all the water out of the dome through the spout. Find work required up to 2-decimal-place accuracy. (The density of water is $\rho = 1000\,\text{kg/m}^3$ and the acceleration due to gravity is $g = 9.81\,\text{m/s}^2$.)



Score: /4