Math 126
Fall 2016
Dr. Lily Yen

# Test 3

Show all your work

Name: _____

Score: ___/41

**No Calculator permitted** in this part. Read the questions carefully. Show all your work and clearly indicate your final answer. Use proper notation.

**Problem 1**: For each sequence or series below, determine if it converges; find the limit or sum if possible, or prove that it diverges to $\infty$, or $-\infty$, or does not exist.

a. $a_n = \dfrac{2 - 5n\sqrt{n}}{3 + n}$. Find the limit of the sequence.

Score: /2

b. $\left\{ \dfrac{\ln n}{n} + \cot\left(\dfrac{n\pi}{n+1}\right) \right\}_{n=1}^{\infty}$. Find the limit of the sequence.

Score: /3

c. $\displaystyle\sum_{n=0}^{\infty} \frac{2(5^n)}{3^{2n}}$.

Score: /3

d. $\displaystyle\sum_{n=2}^{\infty} \frac{2}{n^2-1}$.

Score: /3

e. $\displaystyle\sum_{n=0}^{\infty} \frac{(-10)^n}{n!}$. Determine both convergence and absolute convergence of the series.

Score: /4

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**Calculators permitted** in this part.

**Problem 2**: Consider the infinite series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n}{n^2+1}$$

a. Evaluate the $N$th partial sum for the first four $N$'s. Provide exact answers only.

Score: /2

b. With your calculator, plot the $N$th partial sum for the first 20 $N$'s. You must specify your TI entries.

Score: /2

c. Approximate this infinite series with the 100th partial sum and find the error bound. Provide a 6-decimal place accuracy.

Score: /2

**Problem 3**: For the function $f(x) = e^x + x^2$, set up the integral expression for its arc length for $x \in [1, 4]$. Approximate the integral accurate to 4 decimal places.

Score: /3

**Problem 4**: Use separation of variables to find the general solution of the following differential equation.

$$y' = \frac{\sqrt{1-y^2}}{1+x^2}$$

Score: /3

**Problem 5**: Find the orthogonal trajectories of $y = k\ln(x)$, for arbitrary constants $k$ and positive $x$. Sketch at least three members of each family (the function and its orthogonal trajectory). Label each curve with its parameter value.

Score: /6

**Problem 6**: A stomach flu spread through Capilano University. Let $y(t)$ be the fraction of the population at Cap that has gotten the flu at time $t$ days from first observation. Assume that the rate at which the flu spreads is proportional to the product of the fraction $y$ of the population that has the flu and the fraction $1 - y$ that has not yet contracted the flu.

a. Write down the differential equation satisfied by $y$ in terms of a proportionality constant $k$.

b. Find $k$, assuming that 10 % of the population at Cap got the flu at $t = 0$, and 40 % got the flu at $t = 2$ days.

c. Using the assumption of the previous part, determine when three quarters of the population at Cap will have the flu. Provide a 2-decimal place accuracy.

Score: /8