Math 126
Fall 2015
Dr. Lily Yen

Test 1

Show all your work

Name: _____

Score: ___/50

No Calculator permitted in this part. Read the questions carefully. Show all your work and clearly indicate your final answer. Use proper notation.

Problem 1: Express the limit as a definite integral.

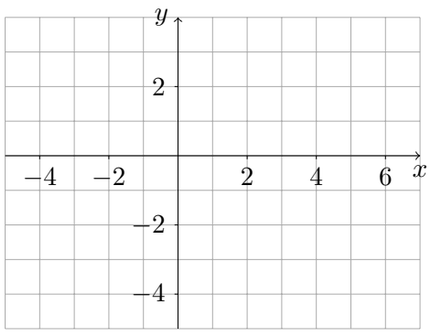
$$\lim_{n\to\infty}\left(\sum_{k=1}^{n}\sin\left(5+\frac{(\pi-3)k}{n}\right)\frac{\pi-3}{n}\right)=$$

Score: /3

Problem 2: Consider the following piece-wise defined function.

$$f(x)=\begin{cases}2-\sqrt{4-(x+2)^2}, & -4\le x<0\\ 2-x, & 0\le x\le 6\end{cases}$$

Draw f in the grid provided and answer the following questions.



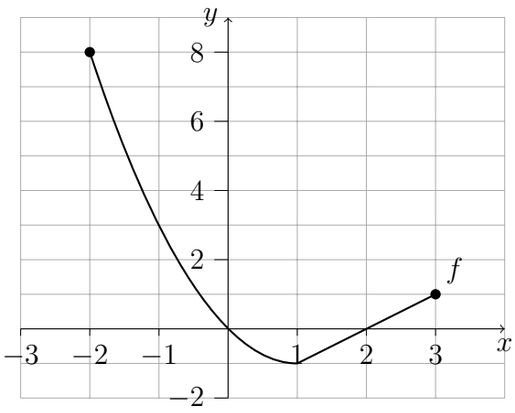
a. $\displaystyle\int_{-4}^{0} f(t)\,dt$

b. $\displaystyle\int_{-2}^{3} |f(x)|\,dx$

c. $\displaystyle\left|\int_{0}^{6} f(u)\,du\right|$

Score: /8

Problem 3: Define $A(x) = \displaystyle\int_{-2}^{x} f(t)\,dt$ for $x \in [-2, 3]$, where the function f is graphed below.



a. Evaluate $A(3) - A(1) =$ _____.

b. Determine the x-value(s) in $[-2, 3]$ where $A(x)$ attains the global maximum.

c. Determine the x-value(s) in $[-2, 3]$ where $A(x)$ attains the global minimum.

d. How do you classify $x = 2$ in the graph of A? Circle all that apply.

INFLECTION PNT. LOCAL MAX LOCAL MIN GLOBAL MAX GLOBAL MIN

Score: /4

Problem 4: Evaluate the following exactly using the Fundamental Theorem of Calculus.

a. Let $g(x) = \displaystyle\int_{2}^{x^2} \frac{1-t}{1+t}\,dt$. Find $g'(x)$.

Score: /1

b. Find $\dfrac{d}{dx}\displaystyle\int_{\cos(x)}^{2x^3} \tan(u)\,du$.

Score: /2

Problem 5: Integrate the following analytically.

a. $\displaystyle\int \frac{(x^2 - \sqrt{x})^2}{x^3}\, dx$

Score: /3

b. $\displaystyle\int (2 - 7x)^6\, dx$

Score: /2

c. $\displaystyle\int \cot^3(5\theta) \csc^2(5\theta)\, d\theta$

Score: /3

d. $\displaystyle\int_{-1}^{1} 3^x e^{7x}\, dx$

Score: /3

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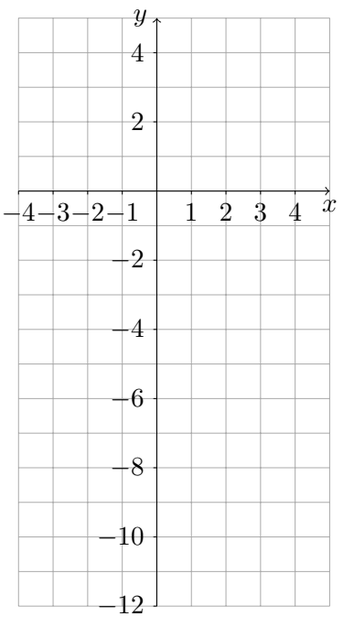
Calculators permitted from here on.

Problem 6: Given the function $f(x) = 4 - x^2$ with a specific partition $P = \{-3, -1, 1.2, 4\}$ and its corresponding sample points $C = \{-2, 0, 3\}$, compute its Riemann sum. You need to draw the graph and its approximating rectangles, then write out each term in the sum before using the calculator to find the answer.



Score: /3

Problem 7: Estimate the distance travelled from 2 seconds after the start of experiment to 6 seconds after the start by a particle with velocity (in m/s)

$$v(t) = \ln(t^2 + 1), \quad t \geq 0.$$

Fill the table below accurate to 4 decimal places.

Approx	$n = 10$	$n = 100$	$n = 200$
L_n			
R_n			

What is the limit of the distance (again accurate to 4 decimal places)?

Score: /4

Problem 8: If the acceleration of an object is given by $a(t) = 2t^2 - 3t$ (in m/s^2), with an initial velocity of 3 m/s. Find the displacement over the interval $[1, 4]$. Does it matter if you do not know the initial position? Explain.

Score: /4

Problem 9: Find the following definite integrals analytically.

a. $\displaystyle\int_0^{\sqrt{2}} x\sqrt{4-x^4}\,dx$

Score: /3

b. $\displaystyle\int_0^5 \frac{dx}{9x^2+25}$

Score: /3

Problem 10: The population of squirrels grows at the rate of $\dfrac{100}{1+t^2}$ squirrels per month since the beginning of the population model.

a. Find the population increase, to the nearest squirrel, from month 3 to month 11.

b. Draw the graph of the population of squirrels for the first year assuming 100 as initial population.

Score: /4