Math 123-02
Summer 2024
Dr. Lily Yen

Midterm Two

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Problem 1: For each of the following, give an exact answer when possible, else round to two decimal places.

a. 16 % of which number is 45?

b. What percentage of 630 is 72?

c. What taxes does one pay on a \$200 purchase? Hint: B.C. has a 5 % GST and a 7 % PST.

If $0.16x = 45$, then $x = \frac{45}{0.16} = 281.25$. $\frac{72}{630} = \frac{4}{35} \approx 0.1143 = 11.43\,\%$.
$200 \times 0.12 = 24$, so 24 dollars in taxes.

Score: /3

Problem 2: Eric purchased some stocks for \$5500 and 30 months later sold the shares for \$8800. Use simple interest to find the annual interest rate expressed as a percent on the earnings of his investment.

The gain in $\$(8800 - 5500) = \$3300$, so $\frac{\$3300}{\$5500} = \frac{3}{5} = 0.60 = 60\,\%$ over 30 months, so $\frac{12}{30} \times 60\,\% = 24\,\%$ per year (simple interest).

Score: /3

Problem 3: Layla takes a \$12 000 loan from the bank at 7 % annual interest rate, to be repaid in 3 years in equal monthly payments. Find her monthly payment and the total interest she pays on this loan. Round to two decimal places.

The total (simple) interest is $3 \times 0.07 \times \$12\,000 = \$2520$, so the total payment (over all three years) is $\$12\,000 + \$2520 = \$14\,520$, so the monthly payment is $\$14\,520 \div 3 \div 12 = \$403.33$.

Score: /3

/9

Problem 4: Evelyn has invested \$28 000 in a savings account that pays 5.5 % annual interest, compounded quarterly (four times a year). How long would it take to double her investment? Round to two decimal places.

Hint: $A = P\left(1 + \dfrac{r}{m}\right)^{mt}$.

$2 \times 28\,000 = 28\,000\left(1 + \frac{0.055}{4}\right)^{4t}$, so $2 = (1.01375)^{4t}$, so
$\log(2) = \log(1.01375^{4t}) = 4t\log(1.01375)$, so

$$t = \frac{\log(2)}{4\log(1.01375)} \approx 12.7 \text{ years}$$

Score: /4

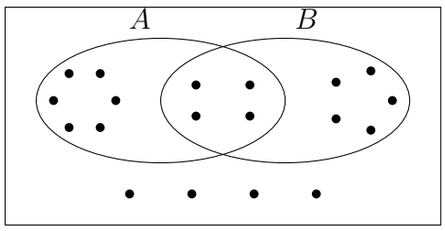
Problem 5: At the end of March, Kelly has a \$1500 balance on their credit card which imposes a 18 % annual interest rate. In April, Kelly incurs \$250 in purchases and makes a payment of \$700. What will their balance be at the end of April? What will the finance charge be in May?

Balance at the end of April is $1500 \times (1 + \frac{0.18}{12}) + 250 - 700 = 1072.50$.
The finance charge for May is $1072.50 \times \frac{0.18}{12} \approx 16.09$.

Score: /4

/8

Problem 6: Dad drew a big rectangle representing a sample space containing two events, A and B. Assuming that the outcomes (as dots) were all equally likely, answer the following questions.



a. $P(A \cap B)$ $\boxed{\frac{4}{19} \approx 21.1\,\%}$

b. $P(A')$ $\boxed{\frac{9}{19} \approx 47.4\,\%}$

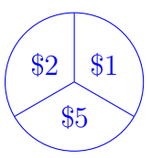
c. $P(A \mid B)$ $\boxed{\frac{4}{9} = 44.4\,\%}$

d. Are A and B disjoint?

$A \cap B \neq \emptyset$, so A and B are not disjoint.

Score: /4

Problem 7: Draw a spinner with three equal sectors labelled \$1, \$2, and \$5. Provide a sample space for spinning the spinner twice. Find the probability that the total after spinning twice is \$3 or less.



+	\$1	\$2	\$5
\$1	2	3	6
\$2	3	4	7
\$5	6	7	10

$P(\text{sum} \leq 3) = \frac{3}{9} = \frac{1}{3} \approx 33.3\,\%$.

Score: /3

Problem 8: A fair die is rolled twice. The following table shows all the possible, equally likely outcomes of this experiment. Use it to find the following probabilities: Use proper notations.

		Second roll					
		1	2	3	4	5	6
1st roll	1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
	2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
	3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
	4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
	5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
	6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

a. Find the probability that the first roll is exactly 3 less than the second roll.

b. Find the probability of getting more than a sum of 10 in two rolls.

$P(X_2 - X_1 = 3) = \frac{3}{36} = \frac{1}{12} \approx 8.3\,\%$. $P(X_1 + X_2 > 10) = \frac{3}{36} = \frac{1}{12} \approx 8.3\,\%$.

Score: /3

/10

Problem 9: The president proposed that all students must take a course in ethics as a requirement for graduation. Three hundred people among faculty members and students were asked about their opinion on this issue. Their choices are summarized in the following table. Answer the following.

	Favour	Oppose	Neutral	Total
Students	40	140	50	230
Faculty	55	10	5	70
Total	95	150	55	300

a. Find the probability that a randomly selected person is in favour of taking ethics as a requirement.

b. Given that the person is a student, find the probability that the person opposes the change.

$P(\text{in favour}) = \frac{95}{300} = \frac{19}{60} = 31.7\,\%.$ $P(\text{oppose}|\text{Student}) = \frac{140}{230} = \frac{14}{23} \approx 60.87\,\%.$

Score: /3

Problem 10: Assume that 7 % of international visitors arriving at the Vancouver International Airport are sick with ARI (Acute respiratory infections). Suppose a test correctly identifies a visitor sick with ARI 92 % of the time. Also assume that the test falsely identifies a healthy visitor as sick with ARI 9 % of the time. If an international visitor tests negative, what is the probability in PERCENT that the visitor is sick with ARI? Round to 4 decimal places.

Draw a probability tree as part of your steps.

$$P(\text{ARI} \mid -) = \frac{P(\text{ARI} \cap -)}{P(-)} = \frac{0.07 \times 0.08}{0.07 \times 0.08 + 0.93 \times 0.91} = 0.006\,574 = 0.6574\,\%$$

Score: /4

/7