Math 123-02
Summer 2024
Dr. Lily Yen

Midterm II

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Problem 1: For each of the following, give an exact answer when possible, else round to two decimal places.

a. What percentage of 360 is 27?

b. 12 % of which number is 25?

c. What taxes does one pay on a $300 purchase? Hint: B.C. has a 5 % and a 7 % PST.

$\frac{27}{360} = \frac{3}{40} = 0.075 = 7.5\,\%$. If $0.12x = 25$, then $x = \frac{25}{0.12} \approx 208.33$. $300 \times 0.12 = 36$, so 36 dollars in taxes.

Score: /3

Problem 2: Beverley purchased some stocks for $6500 and 20 months later sold the shares for $7200. Use simple interest to find the annual interest rate expressed as a percent on the earnings of her investment. Round to two decimal places.

The gain in $\$(7200 - 6500) = \$700$, so $\frac{\$700}{\$6500} = \frac{7}{65} \approx 0.1077 = 10.77\,\%$ over 20 months, so $\frac{12}{20} \times 10.77\,\% = 6.46\,\%$ per year (simple interest).

Score: /3

Problem 3: Jay takes a $16 000 loan from the bank at 6 % annual interest rate, to be repaid in 4 years in equal monthly payments. Find his monthly payment and the total interest he pays on this loan. Round to two decimal places.

The total (simple) interest is $4 \times 0.06 \times \$16\,000 = \$3840$, so the total payment (over all four years) is $\$16\,000 + \$3840 = \$19\,840$, so the monthly payment is $\$19\,840 \div 4 \div 12 = \$413.33$.

Score: /3

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Problem 4: Cassandra has invested \$17000 in a savings account that pays 5.5 % annual interest, compounded semi-annually (twice a year). How long would it take to double her investment? Round to two decimal places.

Hint: $A = P\left(1 + \frac{r}{m}\right)^{mt}$.

$2 \times 17000 = 17000\left(1 + \frac{0.055}{2}\right)^{2t}$, so $2 = (1.0275)^{2t}$, so $\log(2) = \log(1.0275^{2t}) = 2t\log(1.0275)$, so

$$t = \frac{\log(2)}{2\log(1.0275)} \approx 12.8 \text{ years}$$

Score: /4

Problem 5: At the end of March, Sasha has a \$1300 balance on their credit card which imposes a 19 % annual interest rate. In April, Sasha incurs \$250 in purchases and makes a payment of \$600. What will their balance be at the end of April? What will the finance charge be in May?

Balance at the end of April is $1300 \times (1 + \frac{0.19}{12}) + 250 - 600 = 970.58$.
The finance charge for May is $970.58 \times \frac{0.19}{12} \approx 15.37$.

Score: /4

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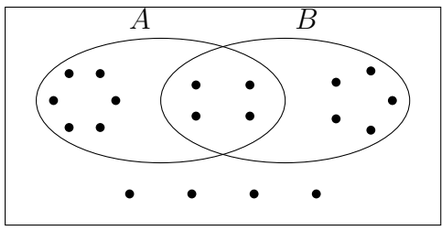
Problem 6: Dad drew a big rectangle representing a sample space containing two events, A and B. Assuming that the outcomes (as dots) were all equally likely, answer the following questions.



a. $P(B')$ $\boxed{\frac{10}{19} \approx 52.6\,\%}$

b. $P(A \cup B)$ $\boxed{\frac{15}{19} \approx 78.9\,\%}$

c. $P(B \mid A)$ $\boxed{\frac{4}{10} = 40.0\,\%}$

d. Are A and B mutually exclusive?

$A \cap B \neq \emptyset$, so A and B are not mutually exclusive.

Score: /4

Problem 7: A fair die is rolled twice. The following table shows all the possible, equally likely outcomes of this experiment. Use it to find the following probabilities: Use proper notations.

		Second roll					
		1	2	3	4	5	6
1st roll	1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
	2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
	3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
	4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
	5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
	6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

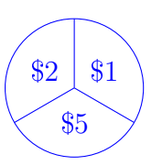
a. Find the probability of getting a sum of less than 4 in two rolls.

b. Find the probability that the first roll is exactly 2 more than the second roll.

$P(\text{sum} < 4) = \frac{3}{36} = \frac{1}{12} \approx 8.3\,\%$. $P(\text{first - second} = 2) = \frac{4}{36} = \frac{1}{9} \approx 11.1\,\%$.

Score: /3

Problem 8: Draw a spinner with three equal sectors labelled \$1, \$2, and \$5. Provide a sample space for spinning the spinner twice. Find the probability that the total after spinning twice is \$5 or more.



+	\$1	\$2	\$5
\$1	2	3	6
\$2	3	4	7
\$5	6	7	10

$P(\text{sum} \geq 5) = \frac{5}{9} \approx 55.6\,\%$.

Score: /3

/10

Problem 9: The president proposed that all students must take a course in ethics as a requirement for graduation. Three hundred people among faculty members and students were asked about their opinion on this issue. Their choices are summarized in the following table. Answer the following.

	Favour	Oppose	Neutral	Total
Students	90	110	30	230
Faculty	45	15	10	70
Total	135	125	40	300

a. Find the probability that a randomly selected person opposes the taking of ethics as a requirement.

b. Given that the person is a member of the faculty, find the probability that the person is in favour of the change.

$P(\text{opposed}) = \frac{125}{300} = \frac{5}{12} \approx 41.7\,\%$. $P(\text{in favour}|\text{faculty}) = \frac{45}{70} = \frac{9}{14} \approx 64.3\,\%$.

Score: /3

Problem 10: Assume that 6 % of international visitors arriving at the Vancouver International Airport are sick with ARI (Acute respiratory infections). Suppose a test correctly identifies a visitor sick with ARI 95 % of the time. Also assume that the test falsely identifies a healthy visitor as sick with ARI 8 % of the time. If an international visitor tests negative, what is the probability in PERCENT that the visitor is sick with ARI? Round to 4 decimal places.

Draw a probability tree as part of your steps.

$$P(\text{ARI} \mid -) = \frac{P(\text{ARI} \cap -)}{P(-)} = \frac{0.06 \times 0.05}{0.06 \times 0.05 + 0.94 \times 0.92} = 0.003\,457 = 0.3457\,\%$$

Score: /4

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