

Midterm Two

Show all your work

Name: _____
 Number: _____
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 Score: ___/30

Problem 1: Answer each question to two decimal places when appropriate. Steps may help you receive part marks.

- a. Find 60% of 8000.

$$0.60 \times 8000 = 4800$$

4800

- b. Convert the fraction $5\frac{2}{3}$ into a percent.

$$5\frac{2}{3} = 5 + \frac{2}{3} = \frac{17}{3} \approx 5.6667 = 566.67\%$$

566.67%

- c. When Brian buys a 300-dollar (ticket price) tent, how much does the store charge

Brian in taxes? Hint: Stores need to charge a 5% GST and a 7% PST.

$$12\% \text{ of } \$300 \text{ is } 0.12 \times 300 = 36.$$

\$36

- d. David treated his mother and grandmother to a Mother's Day Brunch at Queen Elizabeth's Theatre's Browns Social House. He saved \$180 for this occasion to cover a 15% tip and a 12% service tax. What was the maximum they could order to stay within

his budget of \$180?

Say the cost before tip and taxes is $\$x$. Then the tip is $0.15x$, and the tax is $0.12x$, so the table relates the amount of time consumers engage in online shopping per month

- e. The table relates the amount of time consumers engage in online shopping per month with their annual income. Find the probability that a randomly selected consumer spends 0–2 hours per month shopping online.

Annual income	10 h or more	3 h–9 h	0 h–2 h	Total
Above \$60 000	188	179	129	496
\$40 000–\$60 000	147	216	160	523
Below \$40 000	129	188	253	570

$$\frac{129 + 160 + 253}{496 + 523 + 570} = \frac{542}{1589} \approx 34.1\%$$

Score: /5

Problem 2: Solve for the indicated variable.

- a. Solve for t in $A = P(1 + rt)$.

If $A = P(1 + rt)$, then $\frac{A}{P} = 1 + rt$, so $rt = \frac{A}{P} - 1$, so

$$t = \frac{\frac{A}{P} - 1}{r} = \frac{A - P}{Pr}$$

- b. Solve for x in $(3.5)^x = 42$.

If $(3.5)^x = 42$, then $\log((3.5)^x) = \log(42)$, so $x \log(3.5) = \log(42)$, so $x = \frac{\log(42)}{\log(3.5)} \approx 2.984$

- c. Solve for r in $A = P(1 + r/m)^n$.

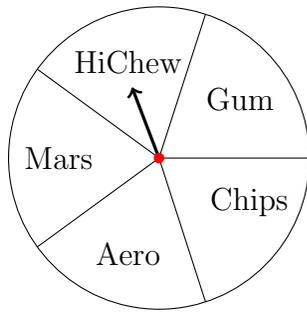
If $A = P(1 + r/m)^n$, then $\frac{A}{P} = (1 + r/m)^n$, so $(A/P)^{1/n} = 1 + r/m$, so $(A/P)^{1/n} - 1 = r/m$, so

$$r = m \left(\left(\frac{A}{P} \right)^{1/n} - 1 \right) = m \sqrt[n]{A/P} - m$$

Score: /5

/10

Problem 3: You are participating in an experiment to spin a spinner with five equal sectors and to roll a cubic die (D6) at the same time. Assume that the pointer never lies on a border line and the die is fair, answer the following questions.



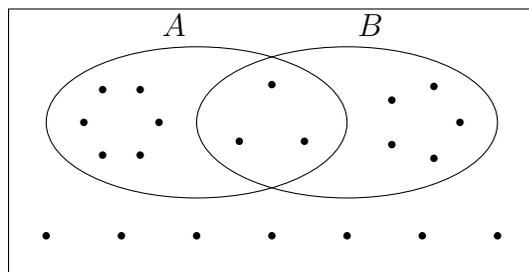
G1	G2	G3	G4	G5	G6
H1	H2	H3	H4	H5	H6
M1	M2	M3	M4	M5	M6
A1	A2	A3	A4	A5	A6
C1	C2	C3	C4	C5	C6

- Next to the spinner above, construct a table for the sample space of the experiment.
- Let X_1 be the random variable for spinning the spinner once. Let X_2 be the random variable for rolling the cubic die once. Find $P((X_1 = \text{Gum}) \cap (X_2 < 2))$.

Since $X_2 < 2$ is the same as $X_2 = 1$, there is only one outcome in $(X_1 = \text{Gum}) \cap (X_2 < 2)$, namely $\{G1\}$, so the probability is $\frac{1}{30} \approx 3.33\%$.

Score: /3

Problem 4: Dad drew a big rectangle representing a sample space containing two events, A and B . Assume that the outcomes (as dots) were all equally likely, answer the following questions.



a. $P(A \cap B)$

$$\frac{3}{21} \approx 14.3\%$$

b. $P(\bar{A})$

$$\frac{12}{21} \approx 57.1\%$$

c. $P(B \cup \bar{A})$

$$\frac{15}{21} \approx 71.4\%$$

d. $P(B | A)$

$$\frac{3}{9} = 33.3\%$$

- e. Are A and B mutually exclusive?

$A \cap B \neq \emptyset$, so A and B are *not* mutually exclusive.

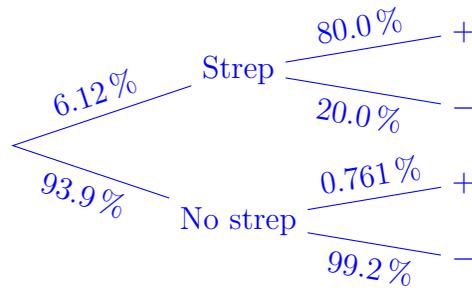
- f. Are A and B independent?

$P(B) = \frac{8}{21}$ and $P(B | A) = \frac{1}{3}$, so $P(B) \neq P(B | A)$, so A and B are *not* independent.

Score: /7

Problem 5: Hamlet woke up with a high fever and a bad cough. When Sam brought him to the doctor, a rapid antigen test from a throat swab sample was performed. From the table below, draw a probability tree with probabilities assigned on the branches. Also find the probability of HAVING no strep throat given a positive test result.

		Streptococcus	
		Have Strep A	No Strep A
Test	+	240	35
Results	-	60	4565



$$P(\text{no strep} \mid +) = \frac{P(\text{no strep but } +)}{P(+)} \approx \frac{93.9\% \times 0.761\%}{6.12\% \times 80.0\% + 93.9\% \times 0.761\%} \approx 12.7\%$$

Score: /4

Problem 6: Suppose Brian's brother purchased a used car for \$10 000 and agreed to pay off the car in 6 monthly instalments of \$2000 each.

- a. Find the total amount of interest charged in this used car loan.

He paid a total of $6 \times \$2000 = \$12\,000$, so the interest was $\$12\,000 - \$10\,000 = \$2000$.

- b. Assume the payments were computed using the add-on interest method; find the annual interest rate applied.

With the add-on method, $I = Prt$, so $r = \frac{I}{Pt} = \frac{\$2000}{\$10\,000 \times 0.5} \approx 40\%$

Score: /3

Problem 7: David's grandparents want to establish a fund for their grandchildren's university education. What lump sum must they deposit at a 2.75% annual interest rate, compounded semi-annually, in order to have \$30 000 in the fund at the end of 18 years?

If

$$\$30\,000 = P \left(1 + \frac{2.75\%}{2} \right)^{18 \times 2} = P(1.013750)^{36},$$

then $P = \frac{\$30\,000}{(1.013750)^{36}} \approx \$18\,348.90$.

Score: /3