

Math 123-02  
 Spring 2026  
 Dr. Lily Yen

# Midterm One

Show all your work

Name: \_\_\_\_\_  
 Number: \_\_\_\_\_  
 Signature: \_\_\_\_\_  
 Score: \_\_\_\_/30

**Problem 1:** Convert  $157_{10}$  into base-8.

$$157 = 2 \times 64 + 3 \times 8 + 5 \times 1 = 235_8$$

Score: /1

**Problem 2:** The following Mayan numeral has three places. Express it as a Hindu-Arabic numeral.



$$= 3 \times (18 \times 20) + 9 \times 20 + 12 = 1272$$

Score: /2

**Problem 3:** The following Kaktovik numeral has 4 places. Find its Hindu-Arabic numeral.



$$= 8 \times 20^3 + 17 \times 20^2 + 6 \times 20 + 14 = 70\,934$$

Score: /2

**Problem 4:** Convert  $23\,789_{10}$  to a Babylonian numeral.

$$23\,789 = 6 \times 60^2 + 36 \times 60 + 29 = \text{𐎶} \text{𐎠𐎠} \text{𐎠𐎠𐎠}$$

Score: /2

**Problem 5:** Fire Horse wants to plant marigolds to fill her rectangular garden. When she lines the seedlings up 7 in a row, she has 5 left over. When she lines them up 8 in a row, she finds her last row short of 3 seedlings to complete a row. Suppose she is planting at least 40 seedlings, find the smallest possible number of marigold seedlings she has.

Say she has  $n$  rows of seven. Then the total number of dragons is  $7n + 5$ . Similarly, if she has  $m$  rows of eight, the total is  $8m - 3$ . Therefore  $7n + 5 = 8m - 3$ , so  $7n + 8 = 8m$ . The solutions to this equation are

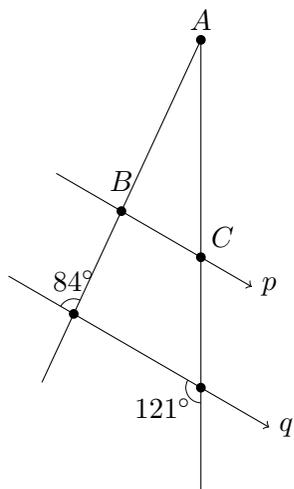
$n$	0	8	16	24	...
$m$	1	8	15	22	...
Total	5	<b>61</b>	117	173	...

Score: /3

/10

**Problem 6:** Lines  $p$  and  $q$  are parallel. Find the measures of INTERIOR angles in  $\triangle ABC$ :

$$\angle A = \boxed{25^\circ}, \angle B = \boxed{96^\circ}, \text{ and } \angle C = \boxed{59^\circ}.$$



The given angle  $84^\circ$  is corresponding to the exterior angle at  $B$ , so the interior angle,  $\angle B = 180^\circ - 84^\circ = 96^\circ$ . Similarly,  $121^\circ$  is a corresponding angle to the exterior angle at  $C$ , so  $\angle C = 180^\circ - 121^\circ = 59^\circ$  form a straight line, so they add up to  $180^\circ$ . The last angle, namely,  $\angle A$  is found by using triangle angle sum, so  $\angle A = 180^\circ - 96^\circ - 59^\circ = 25^\circ$ .

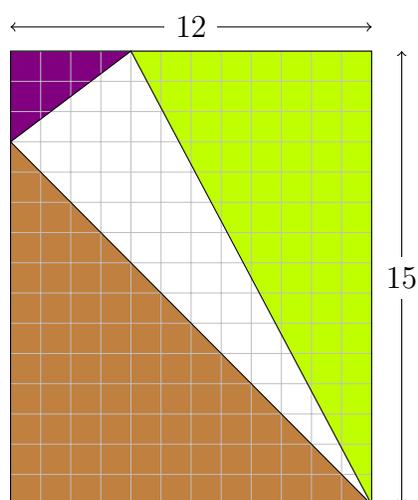
Score: /3

**Problem 7:** Take three different pieces of ropes: 12 cm, 16 cm, and 20 cm. Make three rectangles using each piece as a perimeter for its corresponding rectangle so that two of them make rectangles of the same area while the other makes a rectangle with a larger area.

			
	3	4	1
Perimeter	12 cm	16 cm	20 cm
Area	$9 \text{ cm}^2$	$16 \text{ cm}^2$	$9 \text{ cm}^2$

Score: /3

**Problem 8:** Below is a  $12 \times 15$  grid containing a narrow triangle lying diagonally. Find the exact (no decimal approximation) perimeter and area of the narrow triangle. Show steps.



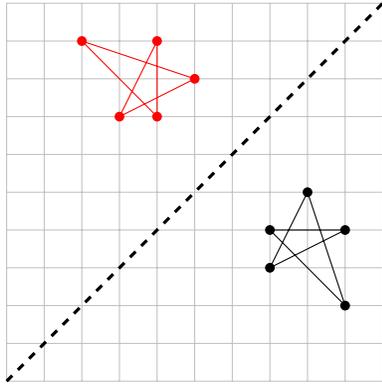
By Pythagorean Theorem, the shortest side has length 5 (from the 3–4–5 triangle), another side is the hypotenuse of an 8–15–17 triangle, thus side length 17. The third side is an isosceles right triangle of side 12, so the hypotenuse is  $12\sqrt{2}$ . All together, the perimeter is  $22 + 12\sqrt{2}$  or  $22 + \sqrt{288}$ .

To find the area of the narrow triangle, subtract the three triangles in the corners from the rectangle:

$$12 \times 15 - \frac{3 \times 4}{2} - \frac{8 \times 15}{2} - \frac{12 \times 12}{2} = 42$$

Score: /4

**Problem 9:** Reflect the given figure along the given diagonal.



Score: /2

**Problem 10:** Signing up with Rogers Home Internet, Mei's family was given six free cell-phone numbers with unlimited domestic calls and texts. If their six numbers all have the same area code (778) and the same first three digits (123), how many different choices do they have for the last four digits?

Only four digits are free, so  $10^4 = 10\,000$  choices.

Score: /2

**Problem 11:** Katharina organized a bag of Haribo's gummy bears according to colour. If Katharina had three times as many red gummy bears as blue ones, and the red and blue gummy bears together equalled half of the rest of the gummy bears, list all the possible total numbers of gummy bears organized by Katharina, starting with the smallest possible number.

Say she had  $x$  blue bears and  $3x$  red ones to a total of  $4x$  red or blue bears. She then had  $8x$  other-coloured bears, to a total of  $12x$  bears.

That is, 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, ...

Score: /2

**Problem 12:** Olove Greek Restaurant has 20 tables, some seat four while the rest seat two. When all tables are occupied to capacity and no tables are put together for a larger party, there are 70 guests. How many tables of each type are in the restaurant?

Suppose there are  $x$  tables for four and  $20 - x$  tables for two. Then  $4x + 2(20 - x) = 70$ , so  $4x + 40 - 2x = 70$ , so  $2x = 30$ , so  $x = 15$ .

That is, 5 two-seaters and 15 four-seaters.

Score: /2

**Problem 13:** Inderpreet is sorting her Canadian bills and coins. Suppose all the bills and coins are different, what are the coins and bills to make \$38.35?

$$\$20 + \$10 + \$5 + \$2 + \$1 + \$0.25 + \$0.10$$

Score: /2