

Midterm One

Show all your work

Name: _____

Number: _____

Signature: _____

Score: ____/30

Problem 1: Convert each of the following into Hindu-Arabic numeral base-10.

a. 210321_4

b. 

c. 

a. $2 \times 4^5 + 4^4 + 3 \times 4^2 + 2 \times 4 + 1 = 2048 + 256 + 48 + 8 + 1 = 2361$.

b. $2 \times 60^2 + 22 \times 60 + 17 = 8537$.

c. $12 \times 20^2 + 9 \times 20 + 4 = 4984$.

Score: /6

Problem 2: Convert 2753_{10} to Mayan numerals.

$$2753 = 7 \times 360 + 11 \times 20 + 13 =$$



Score: /2

Problem 3: We know that we have between 100 and 150 books in our STEM library. If we organize the books in groups of 9s, there are 7 left over; in groups of 7s, 2 books left over. How many books do we have?

Say you have a groups of 9 or b groups of 7. Then total number of books is $9a + 7 = 7b + 2$. For example, $9 \times 1 + 7 = 16 = 7 \times 2 + 2$. Since the $\text{lcm}(9, 7) = 63$, there are $16 + 63 + 63 = 142$ books in the STEM library.

Score: /2

/10

Problem 4: On a Safari Hobby Farm, Farmer Jens has 4 ostriches, 5 zebras, and 3 gazelles. Suppose Farmer Jens needs to bring one of each kind of animals to an elementary school’s fun fair, how many different groups of 3 (namely, an ostrich, a zebra, and a gazelle) are there?

$4 \times 5 \times 3 = 60$ combinations of 3 different kinds of animals.

Score: /2

Problem 5: Sarbjit has 18 coins in her wallet which are either dimes (10-cent coins) or quarters (25-cent coins). She has a total of \$2.25 worth of coins in her wallet. How many quarters does Sarbjit have?

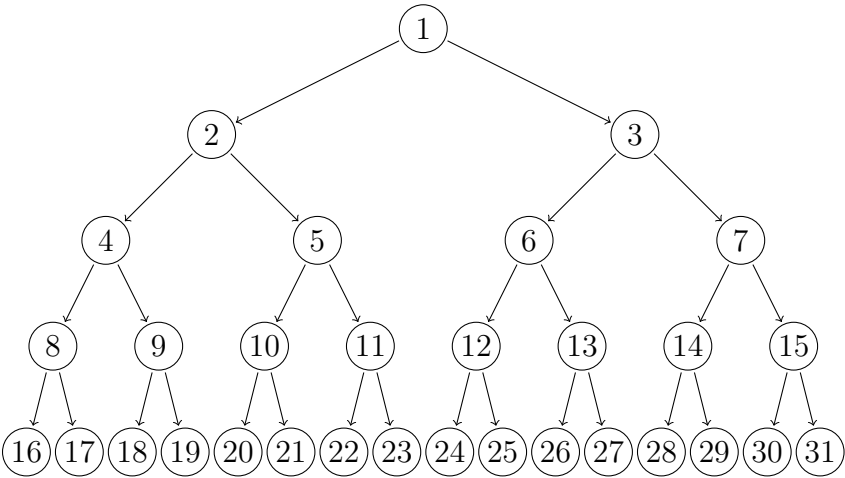
Say she has x dimes and $18 - x$ quarters. Then the value in cents of her coins is

$$10x + 25(18 - x) = 225,$$

so $10x + 450 - 25x = 225$, so $-15x = 225 - 450 = -225$, so $x = 15$. So Sarbjit has 15 dimes and $18 - 15 = 3$ quarters.
Therefore Sarbjit has **three quarters**.

Score: /3

Problem 6: Shown is a binary tree organized in levels: Level 0 has 1 node; level 1 has 2 nodes (labelled 2 and 3); level 2 has 4 nodes (labelled 4, 5, 6, 7).



- How many nodes (circles) are in level 3?
- Which level of the tree contains the label 29?
- If you continue this pattern and draw the next level of the tree, what will the largest label be for the right most node?
- Which level of the tree contains the 4-digit label, 1123? Show your work.

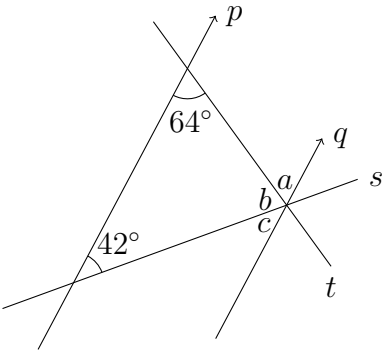
Level 3 contains 8 nodes.
Level 4 contains the label 29.
The next level will be level 5 labelled from 32 to 63, so the rightmost node will be 63.
Continuing with this pattern, we see that

- level 6 has $2^6 = 64$ nodes labelled 64 to 127;
- level 7 has $2^7 = 128$ nodes (128 to 255),
- level 8 has $2^8 = 256$ nodes (256 to 511),
- level 9 has $2^9 = 512$ nodes (512 to 1023),
- level 10 has $2^{10} = 1024$ nodes (1024 to 2047).

Therefore, the label, 1123 is found on Level 10.

Score: /5

Problem 7: Lines p and q are parallel. Lines s and t are transversals. Find the measures of angles a , b , and c .



$c = 42^\circ$, $a = 64^\circ$, and $b = 180^\circ - 42^\circ - 64^\circ = 74^\circ$.

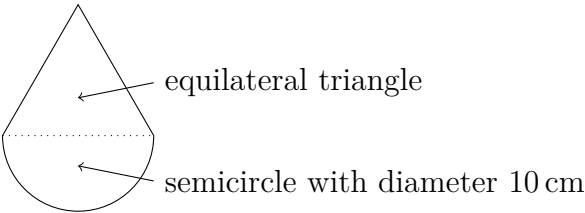
Score: /3

Problem 8: Find the measure of one interior angle of a regular octagon (8-sided).

$(8 - 2) \times 180^\circ / 8 = 135^\circ$.

Score: /1

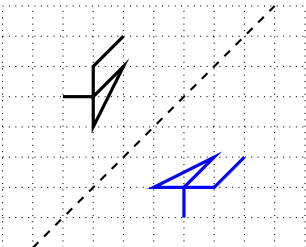
Problem 9: Shown is an equilateral triangle with its base equal to the diameter of a semicircle. If the diameter is 10 cm, find both the perimeter and the area of the figure.



Perimeter is $10 + 10 + 2\pi \times 5/2 = 20 + 5\pi$ centimetres.
 The area of the circle is $\pi 5^2/2 = \frac{25\pi}{2}$ while the triangle area is $\frac{1}{2} \times 10 \times 5\sqrt{3} = 25\sqrt{3}$; so the combined area is $\frac{25\pi}{2} + 25\sqrt{3}$ square cm.

Score: /5

Problem 10: Reflect the given figure along the dashed line.



Score: /1