Math 123-02
Fall 2024
Dr. Lily Yen

Midterm One

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Name: _____
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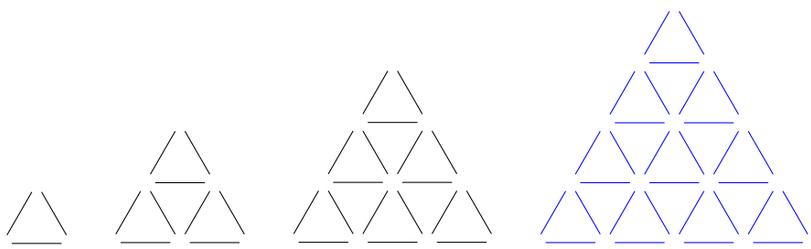
Score: ___/35

Problem 1: A total of 36 handshakes were exchanged at a CapU student union party. Each person shook hands exactly once with each of the others. How many people were present at the party? Hint: A table of values may help.

If the number of people is n, then $\binom{n}{2} = \frac{n(n-1)}{2}$ handshakes are made. Therefore, $\frac{n(n-1)}{2} = 36$, so $n(n-1) = 72$, so $n = 9$.

Score: /2

Problem 2: Shown are pictures of triangles formed by toothpicks: first with one triangle formed by 3 toothpicks; second with a big triangle of sides containing 2 toothpicks each; third with a bigger triangle of sides of length, 3 toothpicks each, and so on.



a. Draw the fourth triangle.

b. Find the number of toothpicks used in the fourth triangle by establishing the pattern from the first.

c. Find a formula expressing the number of toothpicks used in the n-th triangle.

As was discussed in class, the number of toothpicks is

1st	$3(1) = 3$
2nd	$3(1+2) = 9$
3rd	$3(1+2+3) = 18$
4th	$3(1+2+3+4) = 30$
nth	$3(1+2+\cdots+n) = 3 \times \frac{n(n+1)}{2} = \frac{3}{2}n(n+1)$

Score: /6

Problem 3: In a farm with chickens and horses (at least one of each), suppose you count a total of 22 legs, how many of each kind may be on the farm?

Since each chicken has two legs and a horse has four, we can tabulate the number of each below and check the number of legs in each case.

Chickens:	1	3	5	7	9
Horses:	5	4	3	2	1

Suppose in addition, you count 20 eyes in total from the chickens and horses (still with 22 legs), how many of each kind do you have?

From the table, we see that a total of 10 animals giving 20 eyes only happened once, that is, 9 chickens and 1 horse.

Score: /4

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Problem 4: For a project in one of Mei's courses, every student had to write a project proposal choosing three different topics out of 23 listed by the professor. The professor required that the proposed topics be listed in priority order: first, second, and third choice. How many different project proposals containing 3 topics are possible?

There are 23 different topics for the first choice, 22 for the second, 21 for the third, so a total of $23 \times 22 \times 21 = 10\,626$ different project proposals possible.

Score: /2

Problem 5: James and Candice just became friends with Beatrice, and they want to know when her birthday is. Beatrice gives them a list of ten possible dates:

February 3 16
March 15 24
June 1 15 25
September 1 16 24

Beatrice then tells James and Candice separately the month and the day of her birthday, respectively.

James: I don't know when Beatrice's birthday is, but I know that Candice doesn't know either.

Candice: At first I didn't know when Beatrice's birthday is, but I know now.

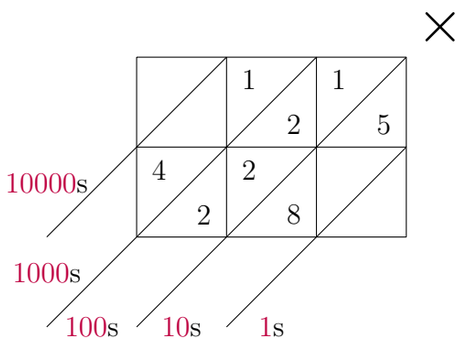
James: I STILL DON'T know when Beatrice's birthday is.

What month is Beatrice's birthday in?

Note that the 3rd and the 25th are unique days in the given list of possible dates. If Candice had been given one of these days, she would know Beatrice's birthday, but James *knows* that Candice doesn't know Beatrice's birthday, so he cannot hold either February or June.
With this information, Candice knows that the month is either March or September. Since she now knows the birthday, it cannot be the 24th. If James held March, he would conclude the birthday must be March 15th. But he does not know, so he must hold **September** and the birthday must be September 1st or 16th.

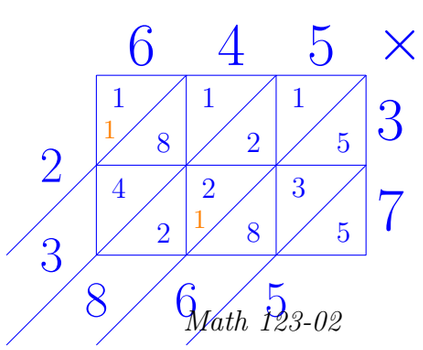
Score: /4

Problem 6: Determine the numbers around the galley to complete the multiplication.



Score: /4

Start with two known cells in a row or in a column to find one-digit common factors; for example, $\gcd(42, 18) = 6$, so 6 must be at the right end of the bottom row with 42 and 18. Similarly, $\gcd(12, 20) = 4$, so 4 is above 6 at the right end of the galley. Note that 2 is also a common factor of 12 and 20, but $20 \div 2 = 10$, a two-digit quotient, not allowed on top of a column in the galley.



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Problem 7: Convert the following into Hindu-Arabic numerals base-10.

a. 

$27 \times 60^2 + 34 \times 60 + 52 = 99\,292$

Score: /2

b. 

$3 \times 20^3 + 7 \times 20^2 + 9 \times 20 + 12 = 26\,992$

Score: /2

Problem 8: Convert the following Hindu-Arabic base-10 numerals into the stated numerals.

a. 1947 into Roman numerals.

MCMXLVII

Score: /2

b. 283 into octal (base-8).

$283 = 4 \times 8^2 + 3 \times 8 + 3 = 433_8$

Score: /2

c. 2968 into Mayan numerals.

$2968 = 8 \times (18 \times 20) + 4 \times 20 + 8,$

Score: /2

Problem 9: Katherina bought a bag of Haribo gummy bears one day. When she grouped all the gummy bears into 25s, she had 4 gummy bears left over. When she grouped them into 20s, she had 9 left over. If her bag contained at least 100 gummy bears, what was the smallest possible number of gummy bears in her bag?

If the total is x, then $x = 25a + 4$ and $x = 20b + 9$, so $25a + 4 = 20b + 9$, so $25a = 20b + 5$, so $5a = 4b + 1$, so

	a	1	5	9	13	17
	b	1	6	11	16	21
Total	$25a + 4 = 20b + 9$	29	129	229	329	429

Score: /3

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