

Math 123-02
Fall 2024
Dr. Lily Yen

Quiz One

Show all your work

Name: _____
Number: _____
Signature: _____
Score: ____/10

Problem 1: Calculate the following:

$$\begin{array}{r} \text{a)} \quad 21 \\ \times 19 \\ \hline 399 \end{array} \quad \begin{array}{r} \text{b)} \quad 61 \\ \times 59 \\ \hline 3599 \end{array} \quad \begin{array}{r} \text{c)} \quad 71 \\ \times 69 \\ \hline 4899 \end{array} \quad \begin{array}{r} \text{d)} \quad 201 \\ \times 199 \\ \hline 39999 \end{array}$$

Compare the answers with the numbers you multiplied. Look for a pattern to help you determine the following two products. 801×799 and $2\,000\,001 \times 1\,999\,999$

Each of the four examples use that $(x+1)(x-1) = x^2 - 1$. In the first example, $x = 20$, so $x+1 = 21$, $x-1 = 19$, and $x^2 = 400$, so $x^2 - 1 = 399$.

Similarly, if $x = 800$, then $801 \times 799 = 800^2 - 1 = 640\,000 - 1 = 639\,999$, and if $x = 2\,000\,000$, then

$$2\,000\,001 \times 999\,999 = 2\,000\,000^2 - 1 = 4\,000\,000\,000 - 1 = 3\,999\,999\,999.$$

Score: /4

Problem 2: In a farm with geese and cows (at least one of each), suppose you count a total of 34 legs, how many of each kind may be on the farm?

Since each goose has two legs and a cow has four, we can tabulate the number of each below and check the number of legs in each case.

Geese:	1	3	5	7	9	11	13	15
Cows:	8	7	6	5	4	3	2	1

Suppose in addition, you count 22 eyes in total from the geese and cows, how many of each kind do you have?

From the table, we see that a total of 11 animals giving 22 eyes only happened once, that is, 5 geese and 6 cows.

Score: /4

Problem 3: Oliver and Vicky just became friends with Yang, and they want to know when his birthday is. Yang gives them a list of ten possible dates:

January 11 17 28
March 7 17
July 15 25
December 7 11 25

Yang then tells Oliver and Vicky separately the month and the day of his birthday, respectively.

Oliver: I don't know when Yang's birthday is, but I know that Vicky doesn't know either.

Vicky: At first I didn't know when Yang's birthday is, but I know now.

Oliver: Then I also know when Yang's birthday is.

When is Yang's birthday?

Note that the 15th and the 28th are unique days in the given list of possible dates. If Vicky had been given one of these days, she would know Yang's birthday, but Oliver *knows* that Vicky doesn't know Kelly's birthday, so he cannot hold either January or July.

With this information, Vicky knows that the month is either March or December. Since she now knows the birthday, it cannot be the 7th. If Oliver held December, he would not know if the birthday is the 11th or the 25th. But he does know the birthday, so it must be **March 17th**.

Score: /2

/10