

Math 123-02
Fall 2024
Dr. Lily Yen

Quiz 1

Show all your work

Name: _____
Number: _____
Signature: _____
Score: ____/10

Problem 1: Calculate the following:

$$\begin{array}{r} \text{a)} \quad 31 \\ \times 29 \\ \hline 899 \end{array} \quad \begin{array}{r} \text{b)} \quad 51 \\ \times 49 \\ \hline 2499 \end{array} \quad \begin{array}{r} \text{c)} \quad 81 \\ \times 79 \\ \hline 6399 \end{array} \quad \begin{array}{r} \text{d)} \quad 101 \\ \times 99 \\ \hline 9999 \end{array}$$

Compare the answers with the numbers you multiplied. Look for a pattern to help you determine the following two products: 901×899 and $1\,000\,001 \times 999\,999$

Each of the four examples use that $(x + 1)(x - 1) = x^2 - 1$. In the first example, $x = 30$, so $x + 1 = 31$, $x - 1 = 29$, and $x^2 = 900$, so $x^2 - 1 = 899$.

Similarly, if $x = 900$, then $901 \times 899 = 900^2 - 1 = 810\,000 - 1 = 809\,999$, and if $x = 1\,000\,000$, then

$$1\,000\,001 \times 999\,999 = 1\,000\,000^2 - 1 = 1\,000\,000\,000\,000 - 1 = 999\,999\,999\,999.$$

Score: /4

Problem 2: In a farm with turkeys and sheep (at least one of each), suppose you count a total of 30 legs, how many of each kind may be on the farm?

Since each turkey has two legs and a sheep has four, we can tabulate the number of each below and check the number of legs in each case.

Turkey:	1	3	5	7	9	11	13
Sheep:	7	6	5	4	3	2	1

Suppose in addition, you count 24 eyes in total from the turkeys and sheep, how many of each kind do you have?

From the table, we see that a total of 12 animals giving 24 eyes only happened once, that is, 9 turkeys and 3 sheep.

Score: /4

Problem 3: Lucas and Florence just became friends with Kelly, and they want to know when her birthday is. Kelly gives them a list of ten possible dates:

January 2 15
June 3 9
August 3 12 21
October 2 9 21

Kelly then tells Lucas and Florence separately the month and the day of her birthday, respectively.

Lucas: I don't know when Kelly's birthday is, but I know that Florence doesn't know either.

Florence: At first I didn't know when Kelly's birthday is, but I know now.

Lucas: Then I also know when Kelly's birthday is.

When is Kelly's birthday?

Note that the 12th and the 15th are unique days in the given list of possible dates. If Florence had been given one of these days, she would know Kelly's birthday, but Lucas *knows* that Florence doesn't know Kelly's birthday, so he cannot hold either January or August.

With this information, Florence knows that the month is either June or October. Since she now knows the birthday, it cannot be the 9th. If Lucas held October, he would not know if

the birthday is the 2nd or the 21st. But he does know the birthday, so it must be **June 3rd**.

Score: /2

/10