Math 108-03
Spring 2026
Dr. Lily Yen

Midterm I

Show all your work

Name: ____________________
Number: ____________________
Signature: ____________________
Score: ____/45

No Calculator allowed in this part.

Problem 1: Determine the following limits analytically showing all steps. Use the symbols DNE, ∞, and $-\infty$ where appropriate.

a. $\displaystyle\lim_{x\to-\infty} \frac{\sqrt{4x^2+3x}}{5x-6} =$ $\boxed{-\frac{2}{5}}$

$$\lim_{x\to-\infty} \frac{\sqrt{4x^2+3x}}{5x-6} = \lim_{x\to-\infty} \frac{\sqrt{x^2(4+3/x)}}{x(5-6/x)} = \lim_{x\to-\infty} \frac{-x\sqrt{4+3/x}}{x(5-6/x)}$$
$$= \lim_{x\to-\infty} \frac{-\sqrt{4+3/x}}{5-6/x} = \frac{-\sqrt{4}}{5} = -\frac{2}{5}$$

Score: /2

b. $\displaystyle\lim_{t\to2^-} \frac{|t-2|}{t^2-4} =$ $\boxed{-\frac{1}{4}}$

$$\lim_{t\to2^-} \frac{|t-2|}{t^2-4} = \lim_{t\to2^-} \frac{-(t-2)}{(t-2)(t+2)} = \lim_{t\to2^-} \frac{-1}{t+2} = \frac{-1}{4}$$

Score: /2

c. $\displaystyle\lim_{x\to3} \frac{\sqrt{x+1}-2}{x-3} =$ $\boxed{\frac{1}{4}}$

$$\lim_{x\to3} \frac{\sqrt{x+1}-2}{x-3} = \lim_{x\to3} \frac{\left(\sqrt{x+1}-2\right)\left(\sqrt{x+1}+2\right)}{(x-3)\left(\sqrt{x+1}+2\right)} = \lim_{x\to3} \frac{\sqrt{x+1}^2-2^2}{(x-3)\left(\sqrt{x+1}+2\right)}$$
$$= \lim_{x\to3} \frac{x+1-4}{(x-3)\left(\sqrt{x+1}+2\right)} = \lim_{x\to3} \frac{x-3}{(x-3)\left(\sqrt{x+1}+2\right)} = \lim_{x\to3} \frac{1}{\sqrt{x+1}+2} = \frac{1}{4}$$

Score: /2

Problem 2: Answer the following using derivative rules. Do NOT simplify.

a. Find the derivative of $h(x) = (x^4 - 3\sqrt{x} + 2\pi)^5 \sec(\sqrt[3]{x})$.

$$h'(x) = 5\left(x^4 - 3\sqrt{x} + 2\pi\right)^4 \left(4x^3 - \frac{3}{2\sqrt{x}}\right)\sec(\sqrt[3]{x})$$
$$+ \left(x^4 - 3\sqrt{x} + 2\pi\right)^5 \sec(\sqrt[3]{x})\tan(\sqrt[3]{x}) \cdot \tfrac{1}{3}x^{-2/3}$$

Score: /2

b. Find $d(g(x))/dx$ where

$$g(x) = \frac{\cot(x^2) - \log(5-2x)}{3^{4-x} + \cos(7x)}$$

$$\frac{d(g(x))}{dx} = \frac{\left(-2x\csc^2(x^2) - \frac{-2}{(5-2x)\ln(10)}\right)(3^{4-x}+\cos(7x)) - (\cot(x^2) - \log(5-2x))(-3^{4-x}\ln(3) - 7\sin(7x))}{(3^{4-x}+\cos(7x))^2}$$

Score: /3

/11

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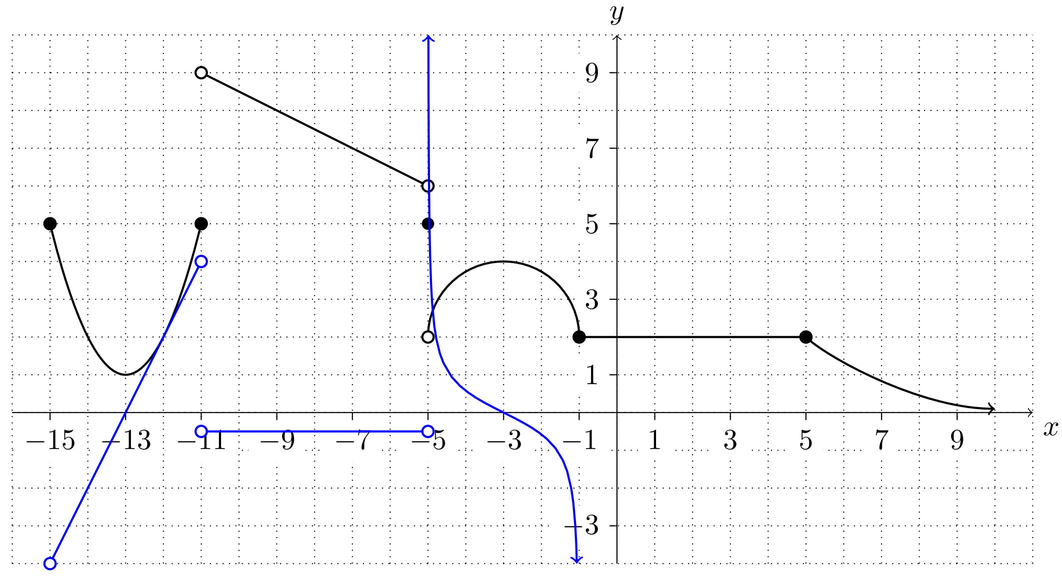
Calculators allowed from here on.

Problem 3: The graph of $y = f(x)$ is shown, piece-wise defined by lines, a parabola, a semi-circle, and a decreasing reciprocal function. Use the graph to answer the questions. Use the symbols DNE, ∞, and $-\infty$ where appropriate.



a. In the domain of the function f on $[-15, \infty)$, list all x where f is discontinuous.

$x = -11, -5$

b. List the x value(s) where f is continuous but not differentiable.

$x = -1, 5$

c. $\lim_{x \to -5^+} f(x) =$ **2**

d. $\lim_{x \to -5^-} f(x) =$ **6**

e. $f(-5) =$ **5**

f. $\lim_{x \to -13} f'(x) =$ **0**

g. $\lim_{h \to 0} \dfrac{f(\pi + h) - f(\pi)}{h} =$ **0**

h. $\lim_{x \to -1^-} \dfrac{f(x) - f(-1)}{x + 1} =$ $-\infty$

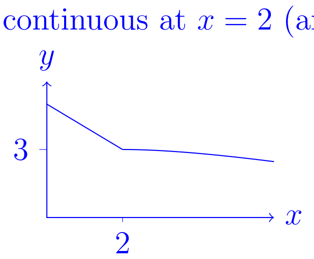
i. In the same grid above, graph $y = f'(x)$ for the x-interval, $(-15, -1)$.

Score: /10

Problem 4: Use the limit definition of continuity to find a value c that makes the piece-wise defined function continuous everywhere. Draw your resulting function to check. From the graph, is the function differentiable at $x = 2$?

$$g(x) = \begin{cases} 5 - x, & x \le 2 \\ \sqrt{2x} - \dfrac{x}{2} + c, & x > 2 \end{cases}$$

Note that $\lim_{x\to 2^-} g(x) = \lim_{x \to 2^-} 5 - x = 3$, that $\lim_{x \to 2^+} g(x) = \lim_{x \to 2^+} \sqrt{2x} - \frac{x}{2} + c = 2 - 1 + c = 1 + c$, and that $f(2) = 3$. Therefore f is continuous at $x = 2$ (and hence everywhere) if $1 + c = 3$, so $c = 2$.



The graph looks like it has a cusp at $(2, 3)$, so g is likely NOT DIFFERENTIABLE and further analysis bears this out.

Score: /4

/14

Problem 5: Use correct notation, show all steps and leave your answer in simplified form.

a. Use the limit definition of the derivative to find the derivative of $f(x) = \sqrt{2x-1}$.

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{2(x+h)-1}-\sqrt{2x-1}}{h} \\ &= \lim_{h\to 0} \frac{\left(\sqrt{2(x+h)-1}-\sqrt{2x-1}\right)\left(\sqrt{2(x+h)-1}+\sqrt{2x-1}\right)}{h\left(\sqrt{2(x+h)-1}+\sqrt{2x-1}\right)} \\ &= \lim_{h\to 0} \frac{(2(x+h)-1)-(2x-1)}{h\left(\sqrt{2(x+h)-1}+\sqrt{2x-1}\right)} = \lim_{h\to 0} \frac{2h}{h\left(\sqrt{2(x+h)-1}+\sqrt{2x-1}\right)} \\ &= \lim_{h\to 0} \frac{2}{\sqrt{2(x+h)-1}+\sqrt{2x-1}} = \frac{2}{2\sqrt{2x-1}} = \frac{1}{\sqrt{2x-1}} \end{aligned}$$

b. Find an equation of the tangent line to f at $x = 5$.

Since $f(5) = \sqrt{9} = 3$ and $f'(5) = \frac{1}{\sqrt{9}} = \frac{1}{3}$, the tangent line is

$$y - 3 = \frac{1}{3}(x-5) \quad \text{or} \quad y = \frac{1}{3}x + \frac{4}{3} \quad \text{or} \quad x - 3y + 4 = 0$$

Score: /5

Problem 6: The distance in metres of an object from a starting point after t seconds is given by $s(t) = 4 + \dfrac{3}{t+1} + t^3$.

a. Find the average velocity of the object over the first 5 seconds.

$$\frac{s(5)-s(0)}{5-0} = \frac{129.5-7}{5} = 24.5\,\text{m/s}$$

Score: /2

b. Use a graphing calculator (TI83, TI83+, TI84-Plus) to set up a table of values to estimate the instantaneous velocity at 5 seconds. Round your answers to 2 decimal places. Specify your Y_1 and Y_2 as part of your steps.

$$s'(5) \approx 74.92\,\text{m/s}$$

Score: /3

/10

Problem 7: Show that the equation $x^2 - x - 1 = \dfrac{1}{1+x}$ has a solution in the interval $(1,2)$ using the Intermediate Value Theorem.

Let $f(x) = x^2 - x - 1 - \frac{1}{1+x}$. Then f is continuous on $(-1, \infty)$, and $f(1) = -\frac{3}{2}$, and $f(2) = \frac{2}{3}$. By the IVT, $f(x) = 0$ for some $x \in (1,2)$.

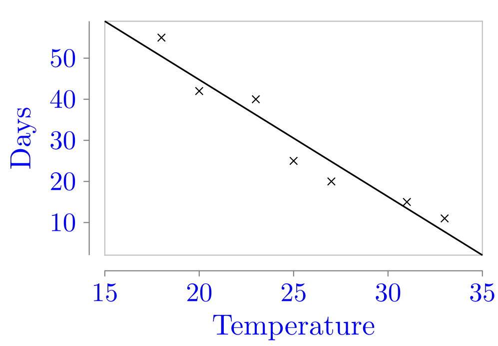
Score: /4

Problem 8: Avocado seeds sprout faster in warmer temperatures. The following table shows groups of seeds and average sprouting time under each temperature.

Temperatures (°C):	20	18	23	25	31	27	33
Average number of Days to sprout:	42	55	40	25	15	20	11

Use the given data to answer the following questions:

a. Draw a scatter plot. Provide dimensions of the window and label your axes according to the sample data.



Score: /4

b. Use linear regression to find a model to fit your plot. Report your model to six decimal places.

$$y = -2.846\,457x + 101.688\,976$$

Score: /1

c. According to your model, what is the time needed for avocado seeds to sprout in 14° Celsius? Comment on the reliability of your answer.

If $x = 14$, then $y \approx 61.8$ days, but extrapolation is dubious—on the other hand, the data does seem to fit a line fairly well.

Score: /1

/10