

Quiz 5

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Problem 1: Consider the following function $f(x) = e^x \cos(x)$. Provide answers for 4 decimal places when appropriate.

- Draw f in the interval $[0, 2\pi]$. Hint: You may need two pictures to show features of f near 0 and further from 0.
- Determine interval(s) where f is decreasing.
- Find all critical numbers (x values), then classify them as local or global extrema with derivative tests.
- Find the absolute maximum point (both coordinates).
- Determine x coordinate(s) of inflection point(s) of f with derivative tests.
- Find interval(s) where f is concave down. Support with derivative tests.

$$f'(x) = e^x \cos(x) - e^x \sin(x) = (\cos(x) - \sin(x))e^x,$$

and since $e^x > 0$ (so never zero), $f'(x) = 0$ when $\cos(x) = \sin(x)$, so when $1 = \frac{\sin(x)}{\cos(x)} = \tan(x)$, so at $\frac{\pi}{4} + n\pi$, $n \in \mathbb{Z}$. Moreover,

$$f''(x) = (-\sin(x) - \cos(x))e^x + (\cos(x) - \sin(x))e^x = -2\sin(x)e^x$$

so $f''(\frac{\pi}{4}) = -\sqrt{2}e^{\pi/4} < 0$, so $(\frac{\pi}{4}, \frac{\sqrt{2}}{2}e^{\pi/4}) \approx (\frac{\pi}{4}, 1.55)$ is a **local maximum**; and $f''(\frac{5}{4}\pi) = \sqrt{2}e^{5\pi/4} > 0$, so $(\frac{5}{4}\pi, -\frac{\sqrt{2}}{2}e^{5\pi/4}) \approx (\frac{5}{4}\pi, -35.9)$ is a **local minimum**. Also, $f(x)$ is **decreasing** between these critical points, so on the interval $(\frac{\pi}{4}, \frac{5}{4}\pi)$.

Now $f(0) = 1$ and $f(2\pi) = e^{2\pi} \approx 535$, so on the closed interval $[0, 2\pi]$, $(\frac{5}{4}\pi, -\frac{\sqrt{2}}{2}e^{5\pi/4}) \approx (\frac{5}{4}\pi, -35.9)$ is a **global minimum**, and $(2\pi, e^{2\pi}) \approx (2\pi, 535)$ is a **global maximum**.

If $f''(x) = 0$, then $\sin(x) = 0$, so $x = n\pi$, $n \in \mathbb{Z}$. Since $f''(x)$ changes sign whenever $\sin(x)$ changes sign, all of these are **inflection points**: $(0, 1)$, $(\pi, -e^\pi) \approx (\pi, -23.1)$, and $(2\pi, e^{2\pi}) \approx (2\pi, 535)$.

From the sign changes in f'' , f is concave down in $(0, \pi)$.

