

Math 108-03
Spring 2026
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Quiz Four

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Problem 1: Use the method of Linear Approximation to approximate $\sqrt[3]{7.85}$.

Let $f(x) = \sqrt[3]{x} = x^{1/3}$. Then $f'(x) = \frac{1}{3}x^{-2/3}$, and

$$\sqrt[3]{7.85} = f(7.85) \approx f(8) + f'(8)(7.85 - 8) = 2 + \frac{1}{12} \times (-0.15) \approx 1.98750$$

Actually $\sqrt[3]{7.85} \approx 1.98742$.

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Problem 2: You and a friend are riding your bikes to a restaurant that you think is east; your friend thinks the restaurant is north. You both leave from the same point, with you riding at 25 km/h east and your friend riding 29 km/h north. After you have travelled 5 km, at what rate is the distance between you changing? Provide 2 decimal places in your answer. Remember to include units.

Let the distances from the common starting point be x and y . Then $\frac{dx}{dt} = 25$ and $\frac{dy}{dt} = 29$. The distance between you and your friend is $z = \sqrt{x^2 + y^2}$, so

$$\frac{dz}{dt} = \frac{d}{dt} \sqrt{x^2 + y^2} = \frac{1}{2\sqrt{x^2 + y^2}} \left(2x \frac{dx}{dt} + 2y \frac{dy}{dt} \right) = \frac{x \frac{dx}{dt} + y \frac{dy}{dt}}{\sqrt{x^2 + y^2}}$$

When $x = 5$, then $t = \frac{1}{5}$, so $y = \frac{29}{5}$, and

$$\frac{dz}{dt} = \frac{5 \cdot 25 + \frac{29}{5} \cdot 29}{\sqrt{5^2 + \left(\frac{29}{5}\right)^2}} = \sqrt{1466} \approx 38.29 \text{ km/h}$$

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Problem 3: Given the following implicitly defined function, first determine $\frac{dy}{dx}$; second, find the equation of the tangent line at the point where $x = 2$.

$$x^3 + 19 + 8y = y^3 + 2x^2y$$

$$3x^2 + 8y' = 3y^2y' + 4xy + 2x^2y'$$

so $3x^2 - 4xy = 3y^2y' + 2x^2y' - 8y' = (3y^2 + 2x^2 - 8)y'$, so

$$\frac{dy}{dx} = y' = \frac{3x^2 - 4xy}{3y^2 + 2x^2 - 8}$$

If $x = 2$, then $2^3 + 19 + 8y = y^3 + 2 \times 2^2y$, so $27 + 8y = y^3 + 8y$, so $27 = y^3$, so $y = 3$.
If $x = 2$ and $y = 3$, then

$$m = \left. \frac{dy}{dx} \right|_{(x,y)=(2,3)} = \frac{3 \times 2^2 - 4 \times 2 \times 3}{3 \times 3^2 + 2 \times 2^2 - 8} = -\frac{4}{9}$$

so the tangent line is

$$y - 3 = -\frac{4}{9}(x - 2) \quad \text{or} \quad y = -\frac{4}{9}x + \frac{35}{9} \quad \text{or} \quad 4x + 9y - 35 = 0$$

Score: /4