Math 108-03 Spring 2026 Dr. Lily Yen	# Quiz 4 Show all your work	Name: ________ Number: ________ Signature: ________ Score: ___/10

Problem 1: Given the following implicitly defined function, first determine $\frac{dy}{dx}$; second, find the equation of the tangent line at the point where $x = 2$.

$$x^3 + 19 + 8y = y^3 + 2x^2y$$

$$3x^2 + 8y' = 3y^2y' + 4xy + 2x^2y'$$

so $3x^2 - 4xy = 3y^2y' + 2x^2y' - 8y' = (3y^2 + 2x^2 - 8)y'$, so

$$\frac{dy}{dx} = y' = \frac{3x^2 - 4xy}{3y^2 + 2x^2 - 8}$$

If $x = 2$, then $2^3 + 19 + 8y = y^3 + 2 \times 2^2y$, so $27 + 8y = y^3 + 8y$, so $27 = y^3$, so $y = 3$.
If $x = 2$ and $y = 3$, then

$$m = \left.\frac{dy}{dx}\right|_{(x,y)=(2,3)} = \frac{3 \times 2^2 - 4 \times 2 \times 3}{3 \times 3^2 + 2 \times 2^2 - 8} = -\frac{4}{9}$$

so the tangent line is

$$y - 3 = -\frac{4}{9}(x - 2) \quad \text{or} \quad y = -\frac{4}{9}x + \frac{35}{9} \quad \text{or} \quad 4x + 9y - 35 = 0$$

Score: /4

Problem 2: You and a friend are riding your bikes to a restaurant that you think is east; your friend thinks the restaurant is north. You both leave from the same point, with you riding at 27 km/h east and your friend riding 31 km/h north. After you have travelled 9 km, at what rate is the distance between you changing? Provide 2 decimal places in your answer. Remember to include units.

Let the distances from the common starting point be x and y. Then $\frac{dx}{dt} = 27$ and $\frac{dy}{dt} = 31$.
The distance between you and your friend is $z = \sqrt{x^2 + y^2}$, so

$$\frac{dz}{dt} = \frac{d}{dt}\sqrt{x^2 + y^2} = \frac{1}{2\sqrt{x^2 + y^2}}\left(2x\frac{dx}{dt} + 2y\frac{dy}{dt}\right) = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{\sqrt{x^2 + y^2}}$$

When $x = 9$, then $t = \frac{1}{3}$, so $y = \frac{31}{3}$, and

$$\frac{dz}{dt} = \frac{9 \cdot 27 + \frac{31}{3} \cdot 31}{\sqrt{9^2 + \left(\frac{31}{3}\right)^2}} = 13\sqrt{10} \approx 41.11\,\text{km/h}$$

Score: /3

Problem 3: Use the method of Linear Approximation to approximate $\sqrt[3]{7.89}$.

Let $f(x) = \sqrt[3]{x} = x^{1/3}$. Then $f'(x) = \frac{1}{3}x^{-2/3}$, and

$$\sqrt[3]{7.89} = f(7.89) \approx f(8) + f'(8)(7.89 - 8) = 2 + \frac{1}{12} \times (-0.11) \approx 1.990\,83$$

Actually $\sqrt[3]{7.89} \approx 1.990\,79$.

Score: /3