

Worked examples on reliability

Lily Yen

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Example 1 Suppose buildings collapse due to one of several errors, C_1, C_2, \dots, C_m . For example

- C_1 Poor design (underestimate load; underestimate wind stress; ...)
- C_2 Poor construction (low grade materials; insufficient controls; gross error; ...)
- C_3 A combination of C_1 and C_2
- C_4 Other (non-assignable) causes.

From either past studies (actual percentages of buildings that have the above problems) or the subjective beliefs of expert consultants, one may determine $P(C_i)$, the *prior probabilities*.

Also, suppose that previous experience (or, again, perhaps experts' opinion) determine what the probabilities of collapse would be given each of the above cases; that is, the values of $P(B | C_i)$ are known, where B is "building collapse." These conditional probabilities are sometimes called *risk factors*.

Table 1 summarizes these known probabilities.

1. Calculate the posterior probabilities.
2. What is the chance of finding an assignable cause for the collapse?
3. What is the most likely assignable cause for the collapse?
4. How likely is it that the collapse was caused by more than one assignable cause?

| Cause C_i | Prior probability $P(C_i)$ | Risk factor $P(B C_i)$ | Posterior probability $P(C_i B)$ |
|----------------|-------------------------------|-----------------------------|---------------------------------------|
| C_1 | 0.00050 | 0.10 | |
| C_2 | 0.00010 | 0.20 | |
| C_3 | 0.00001 | 0.40 | |
| C_4 | 0.99939 | 0.0001 | |

Table 1: Causes of building collapse

Example 2 A system consists of several components of type a , b , and c . Let A denote the event that a component a fails, and likewise for B and C . Suppose that $P(A) = 0.05$, $P(B) = 0.02$, $P(C) = 0.03$, and that all components fail or succeed independently. Compute the reliability, that is the probability of success, for each of the following systems.

