Math 336
Spring 2024
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Midterm

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Number: ______________
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Score: ___/30

Problem 1: A simple graph (no loops, no multiple edges, no directed edges) on n vertices has a corresponding adjacency matrix of dimension $n \times n$.

a. What are the entries on the diagonal?

b. How many bits are required to store the adjacency matrix?

c. Prove that $\dfrac{n^2-n}{2} = \Theta(n^2)$.

a. The entries on the diagonal are all 0 (since there are no loops).

b. The matrix is symmetric, so only the entries above the diagonal need to be stored. Each entry is a single bit (0 or 1), so the matrix requires $(n-1)+(n-2)+\cdots+1 = \frac{1}{2}(n-1)n = \frac{n^2-n}{2}$ bits.

c. Clearly $\frac{n^2-n}{2} < \frac{n^2}{2} = \frac{1}{2}n^2$. Moreover, if $n > 2$, then $n^2 - 2n > 0$, so $2n^2 - 2n > n^2$, so by dividing both sides of the inequality by 4, we get $\frac{n^2-n}{2} > \frac{1}{4}n^2$. Therefore, if $n > 2$,

$$\frac{1}{4}n^2 < \frac{n^2-n}{2} < \frac{1}{2}n^2,$$

so $\frac{n^2-n}{2} = \Theta(n^2)$.

Score: /5

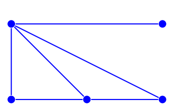
Problem 2: For each given degree sequence of a simple graph, either draw the graph or explain why such a graph does not exist.

a. $0, 1, 2, 3, 4$

b. $1, 2, 2, 3, 4$

c. A connected simple graph of degree sequence $1, 1, 1, 1, 2, 2$.

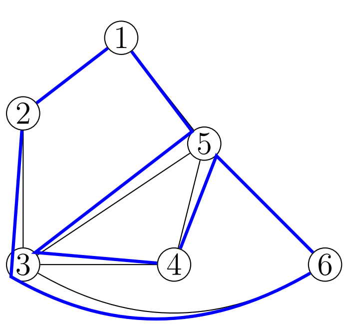
a. The graph has 5 vertices, so the vertex of degree 4 must be connected to every other vertex. Since one vertex has degree 0, this graph is not possible.

b. 

c. A connected graph with 6 vertices must have at least 5 edges (from a tree), so degree sum at least $2 \times 5 = 10$, but $1+1+1+1+2+2 = 8 < 10$, so the graph is impossible.

Score: /5

Problem 3: Does the following graph contain an Eulerian cycle? If so, list the vertices of traversal. If not, explain why not.



Since all the degrees are even, the graph does contain a Eulerian cycle. One such is shown.

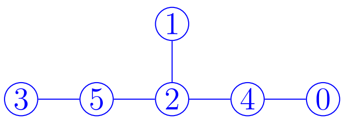
Score: /3

/13

Problem 4: Given the second row of an extended Prüfer code, determine the first and draw the corresponding labelled tree.

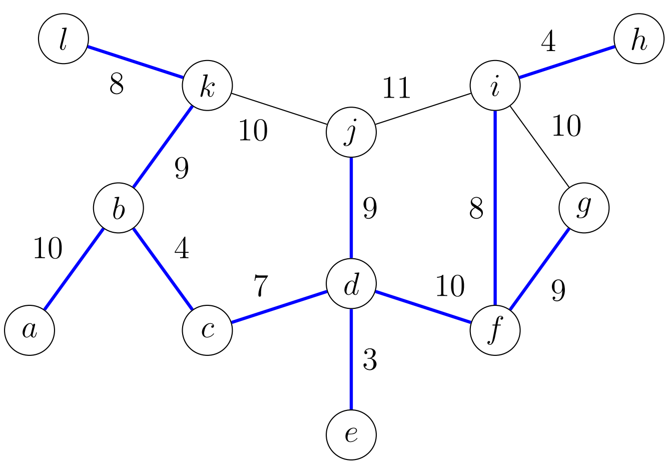
$$2, 5, 2, 4, 0$$

1	3	5	2	4
2	5	2	4	0



Score: /5

Problem 5: Use one of the minimum spanning tree algorithms to find a minimum spanning tree of the following graph. List clearly the order of choice with its corresponding cost and summarize by stating the minimum cost.



Kruskal:

Edge	Cost
e–d	3
b–c	4
h–i	4
c–d	7
f–i	8
k–l	8
b–k	9
d–j	9
f–g	9
d–f	10
a–b	10
Total	81

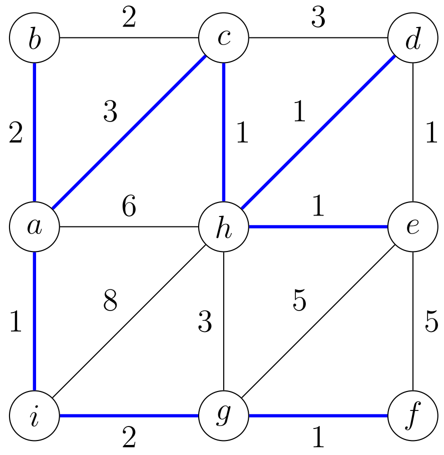
Alternatively, Prim's algorithm begins with vertex a. Then add:

Vertex	Cost
b	10
c	4
d	7
e	3
k	9
l	8
j	9
f	10
i	8
h	4
g	9
Total	81

Score: /5

/10

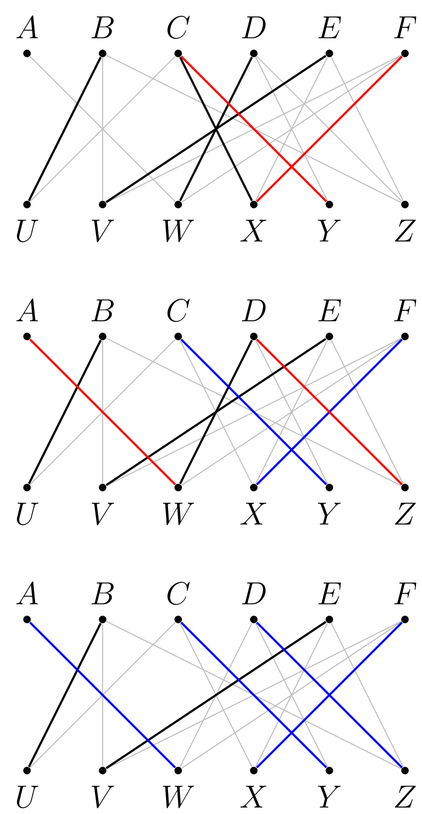
Problem 6: Use Dijkstra's algorithm to find a shortest paths tree from vertex a on the following graph. Track your iterations in a table with vertices for column headings.



Iteration	a	b	c	d	e	f	g	h	i
0	[0]	2	3					6	1
1	[0]	2	3				3	6	[1]
2	[0]	[2]	3				3	6	[1]
3	[0]	[2]	[3]	6			3	4	[1]
4	[0]	[2]	[3]	6	8	4	[3]	4	[1]
5	[0]	[2]	[3]	5	5	4	[3]	[4]	[1]
6	[0]	[2]	[3]	5	5	[4]	[3]	[4]	[1]
7	[0]	[2]	[3]	[5]	[5]	[4]	[3]	[4]	[1]

Score: /5

Problem 7: Demonstrate the *augmenting path* algorithm for finding a perfect matching in the following bipartite graph by carrying out two iterations.



Score: /2

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