

Math 336
Spring 2024
Dr. Lily Yen

Midterm

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Name: _____
Number: _____
Signature: _____
Score: ____/30

Problem 1: A simple graph (no loops, no multiple edges, no directed edges) on n vertices has a corresponding adjacency matrix of dimension $n \times n$.

- What are the entries on the diagonal?
- How many bits are required to store the adjacency matrix?
- Prove that $\frac{n^2 - n}{2} = \Theta(n^2)$.

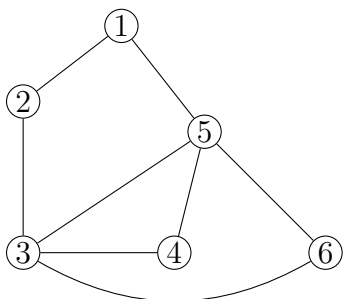
Score: /5

Problem 2: For each given degree sequence of a simple graph, either draw the graph or explain why such a graph does not exist.

- 0, 1, 2, 3, 4
- 1, 2, 2, 3, 4
- A connected simple graph of degree sequence 1, 1, 1, 1, 2, 2.

Score: /5

Problem 3: Does the following graph contain an Eulerian cycle? If so, list the vertices of traversal. If not, explain why not.



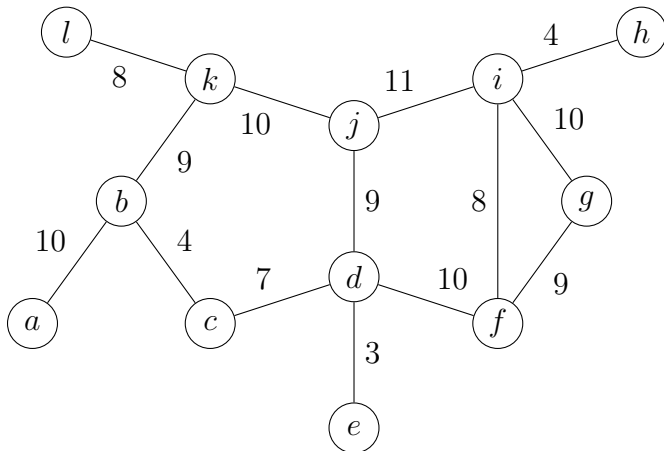
Score: /3

Problem 4: Given the second row of an extended Prüfer code, determine the first and draw the corresponding labelled tree.

2, 5, 2, 4, 0

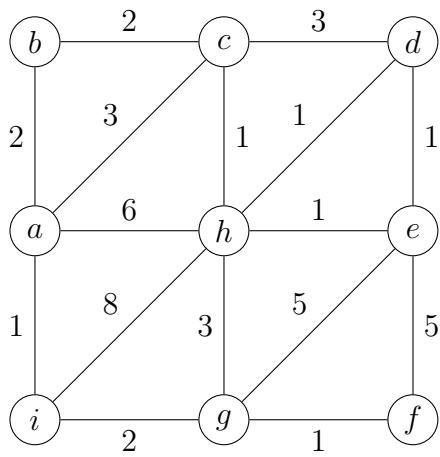
Score: /5

Problem 5: Use one of the minimum spanning tree algorithms to find a minimum spanning tree of the following graph. List clearly the order of choice with its corresponding cost and summarize by stating the minimum cost.



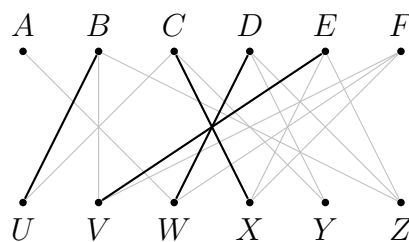
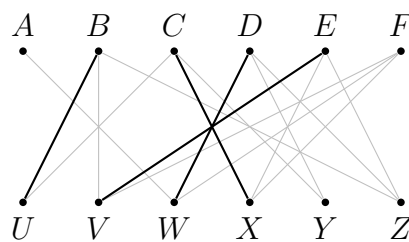
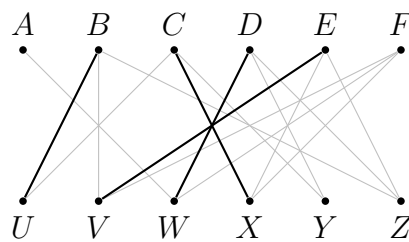
Score: /5

Problem 6: Use Dijkstra's algorithm to find a shortest paths tree from vertex a on the following graph. Track your iterations in a table with vertices for column headings.



Score: /5

Problem 7: Demonstrate the *augmenting path* algorithm for finding a perfect matching in the following bipartite graph by carrying out two iterations.



Score: /2