Math 336
Spring 2024
Dr. Lily Yen

Quiz 5

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Problem 1: Use the matrix representation to rewrite both the following problem and its dual. (One mark each)

$$\begin{array}{llll} \text{Maximize} & & Z = 2x_1 + 7x_2 + 4x_3 \\ \text{subject to} & x_1 + 2x_2 + x_3 & \le 10, \\ & 3x_1 + 3x_2 + 2x_3 & \le 12, \\ \text{and} & x_1, \quad x_2, \quad x_3 \ge 0. \end{array}$$

Determine which model is easier to solve graphically and do so. (Four marks)

Use the simplex method in tabular form (provided on page 2) to solve the other model and check that optimal solutions yield the same value in both primal and dual problems. (Four marks)

Primal: Maximize

$$Z = \begin{bmatrix} 2, & 7, & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

subject to

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \le \begin{bmatrix} 10 \\ 12 \end{bmatrix}, \qquad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \ge \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

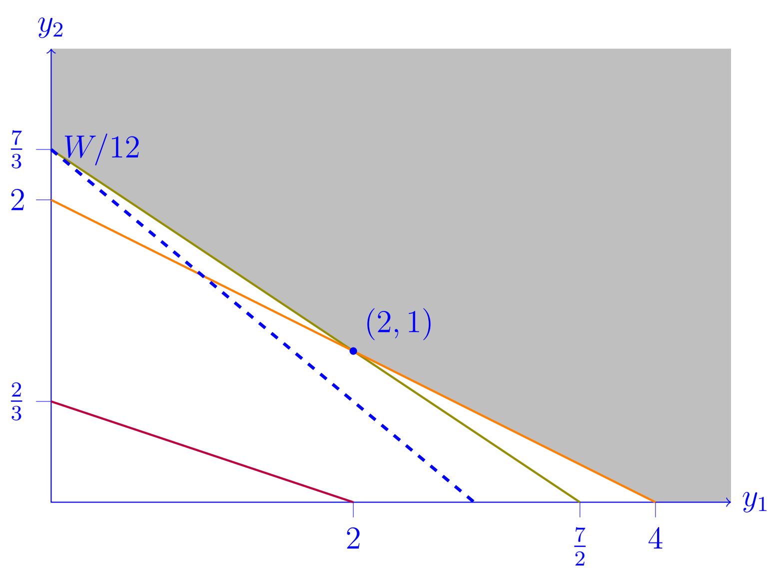
Dual: Minimize

$$W = \begin{bmatrix} y_1, & y_2 \end{bmatrix} \begin{bmatrix} 10 \\ 12 \end{bmatrix}$$

subject to

$$\begin{bmatrix} y_1, & y_2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 3 & 3 & 2 \end{bmatrix} \ge \begin{bmatrix} 2 \\ 7 \\ 4 \end{bmatrix}, \qquad \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \ge \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The dual problem is easier to solve graphically since it is two dimensional while the primal problem is three dimensional.
The (dual) objective equation is $W = 10y_1 + 12y_2$, so $y_2 = -\frac{5}{6}y_1 + \frac{W}{12}$. The constraints are $y_1 + 3y_2 \ge 2$, $2y_1 + 3y_2 \ge 7$, and $y_1 + 2y_2 \ge 4$, so $y_2 \ge -\frac{1}{3}y_1 + \frac{2}{3}$, $y_2 \ge -\frac{2}{3}y_1 + \frac{7}{3}$, and $y_2 \ge -\frac{1}{2}y_1 + 2$. Thus the feasible region is



It.	B.V.	Eq	Z	x_1	x_2	x_3	x_4	x_5	RHS	Ratio
0	Z	(0)	1	-2	-7	-4	0	0	0	
	x_4	I	0	1	2	1	1	0	10	5
	x_5	II	0	3	3	2	0	1	12	4
(0) + 7II 1	Z	(0)	1	5	0	$\frac{2}{3}$	0	$\frac{7}{3}$	28	
I − 2II	x_4	I	0	-1	0	$-\frac{1}{3}$	1	$-\frac{2}{3}$	2	
	x_2	II	0	1	1	$\frac{2}{3}$	0	$\frac{1}{3}$	4	
2										
3										