Math 336
Spring 2024
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Quiz 3

Show all your work

Name: ____________

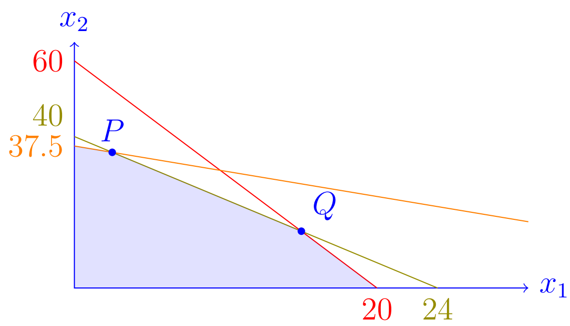
Number: ____________

Signature: ____________

Score: ___/10

Problem 1: Use the graphical method to find all optimal solutions for the following model:

$$\begin{array}{llr} \text{Maximize} & Z = 500x_1 + 300x_2 & x_2 = -\frac{5}{3}x_1 + \frac{1}{300}Z \\ \text{subject to} & 15x_1 + 5x_2 \le 300, & x_2 \le -3x_1 + 60 \\ & 10x_1 + 6x_2 \le 240, & x_2 \le -\frac{5}{3}x_1 + 40 \\ & 8x_1 + 12x_2 \le 450, & x_2 \le -\frac{2}{3}x_1 + \frac{75}{2} \\ \text{and} & x_1, \quad x_2 \ge 0. & \end{array}$$



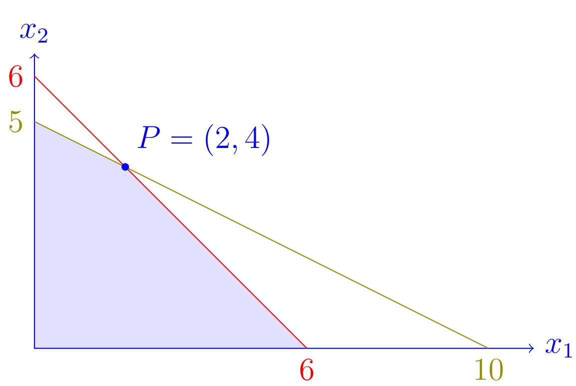
To find P, solve $-\frac{2}{3}x_1 + \frac{75}{2} = -\frac{5}{3}x_1 + 40$, which yields $x_1 = \frac{5}{2}$. To find Q, solve $-3x_1 + 60 = -\frac{5}{3}x_1 + 40$, which yields $x_1 = 15$.
Since $Z = 500x_1 + 300x_2$ and $10x_1 + 6x_2 \le 240$ have the same slope $(-\frac{5}{3})$, all the infinitely many points on the line between $P = (\frac{5}{2}, \frac{215}{6})$ and $Q = (15, 15)$ are optimal.

Score: /3

Problem 2: Consider the following problem, where the value of c_1 has not yet been ascertained.

$$\begin{array}{llr} \text{Maximize} & Z = c_1x_1 + x_2 & x_2 = -c_1x_1 + Z \\ \text{subject to} & x_1 + x_2 \le 6, & x_2 \le -x_1 + 6 \\ & x_1 + 2x_2 \le 10, & x_2 \le -\frac{1}{2}x_1 + 5 \\ \text{and} & x_1, \quad x_2 \ge 0. & \end{array}$$

Use graphical analysis to determine the optimal solution(s) for (x_1, x_2) for the various possible values of $c_1 \in \mathbb{R}$.



To find P , solve $-x_1+6 = -\frac{1}{2}x_1+5$, which yields $x_1 = 2$.
The slope of $x_2 = -c_1x_1 + Z$ is $-c_1$, so you have five cases:

If $c_1 < \frac{1}{2}$, then the optimal solution is $(x_1, x_2) = (0, 5)$.
If $c_1 = \frac{1}{2}$, then any point on the line between $(0, 5)$ and $P = (2, 4)$ is optimal.
If $\frac{1}{2} < c_1 < 1$, then the optimal solution is $(x_1, x_2) = P = (2, 4)$.
If $c_1 = 1$, then any point on the line between $P = (2, 4)$ and $(6, 0)$ is optimal.
If $c_1 > 1$, then the optimal solution is $(x_1, x_2) = P = (6, 0)$.

Score: /3

/6

Problem 3: Capilano University Heavy Metal Company plans to blend a new alloy of 40 % tin, 35 % zinc, and 25 % lead from several available alloys having the following compositions. The company wants to determine the proportions of these alloys that should be blended to produce the new alloy at a minimum cost. Formulate a linear programming model for this problem.

	Alloy				
	1	2	3	4	5
Percentage of tin	60	25	45	20	50
Percentage of zinc	10	15	45	50	40
Percentage of lead	30	60	10	30	10
Cost ($/kg)	47	44	55	51	57

Let x_i (in kg) be the amount if alloy i is used.
Then minimize (in dollars)

$$Z = 47x_1 + 44x_2 + 55x_3 + 51x_4 + 57x_5$$

subject to

$$\begin{aligned} 0.6x_1 + 0.25x_2 + 0.45x_3 + 0.2x_4 + 0.5x_5 &= 0.4(x_1 + x_2 + x_3 + x_4 + x_5) \\ 0.1x_1 + 0.15x_2 + 0.45x_3 + 0.5x_4 + 0.4x_5 &= 0.35(x_1 + x_2 + x_3 + x_4 + x_5) \\ 0.3x_1 + 0.6x_2 + 0.1x_3 + 0.3x_4 + 0.1x_5 &= 0.25(x_1 + x_2 + x_3 + x_4 + x_5) \end{aligned}$$

and $x_1, x_2, x_3, x_4, x_5 \geq 0$.

Score: /4