Math 336
Spring 2024
Dr. Lily Yen

Quiz 1

Show all your work

Name: ____________

Number: ____________

Signature: ____________

Score: ___/10

Problem 1: True or false. Prove your claim.

a. $\cos(x) = \Omega(1)$.

b. $\sum_{j \le n} 2 \sim 3n$.

a. True: Choose $\epsilon = \frac{1}{2}$ and $x_j = 2j\pi$. Then, for all j, $|\cos(x_j)| = |\cos(2j\pi)| = 1$ and $\epsilon \cdot 1 = \frac{1}{2}$, so $|\cos(x_j)| > \epsilon \cdot 1$. Therefore $\cos(x) = \Omega(1)$.

b. False: Note that $\sum_{j \le n} 2 = \sum_{j=1}^{n} 2 = 2n$. Therefore

$$\lim_{n\to\infty} \frac{\sum_{j\le n} 2}{3n} = \lim_{n\to\infty} \frac{2n}{3n} = \lim_{n\to\infty} \frac{2}{3} = \frac{2}{3} \neq 1,$$

so $\sum_{j \le n} 2 \not\sim 3n$

Score: /4

Problem 2: Show that $\sum_{j=1}^{n} j = \Theta(n^2)$.

Note that $\sum_{j=1}^{n} j = \frac{1}{2}n(n+1)$. If $n > 1$, then $\frac{1}{2}n(n+1) > \frac{1}{2}n(n) = \frac{1}{2}n^2$ and $\frac{1}{2}n(n+1) < \frac{1}{2}n(n+n) = \frac{1}{2}n(2n) = n^2$. Therefore

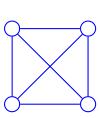
$$\frac{1}{2}n^2 < \sum_{j=1}^{n} j < n^2,$$

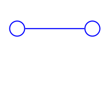
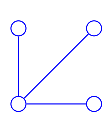
so $\sum_{j=1}^{n} j = \Theta(n^2)$ (with ($c_1 = \frac{1}{2}$ and $c_2 = 1$).

Score: /5

Problem 3: Is it true that if 4 students gather for a Graph Optimization party at Lily's office, there is always a student who knows an even number of other students?

No: If all the students know each other, then every student knows three other students (and 3 is not even).



Or another example:  or 

Score: /1