

Math 205  
Spring 2016  
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# Midterm 2

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Name: \_\_\_\_\_

Score: \_\_\_\_/48

**Problem 1:** The library at Capilano University has a copy of Math 205 text on two-hour reserve. Let  $X$  denote the amount of time the text is actually checked out, and suppose the cdf is

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{x^2}{4}, & 0 \leq x < 2 \\ 1, & 2 \leq x \end{cases}$$

Use the cdf to answer the following.

- a. Find the probability that the text is checked out for more than half an hour but less than an hour.

Score: /1

- b. Find the median check out duration.

Score: /1

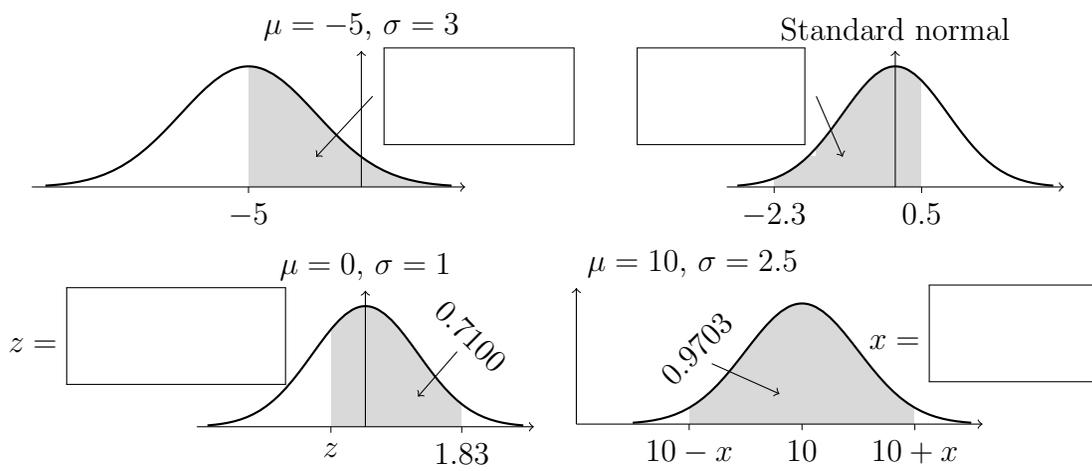
- c. Find the probability density function of  $X$ , that is,  $f(x)$ .

Score: /2

- d. Find the expected value and standard deviation of  $X$ .

Score: /4

**Problem 2:** Identify each missing value in the following normal curves. Provide 4-decimal place accuracy unless the answer is expressed by a fraction exactly.



Score: /4

**Problem 3:** Suppose 90% of all B.C. drivers regularly wear a seat belt while operating a vehicle. A random sample of 500 drivers is selected. Find the following probabilities.

- a. Between 300 and 350 (inclusive) of the drivers in the sample regularly wear a seat belt.

Score: /3

- b. Fewer than 450 of those in the sample regularly wear a seat belt.

Score: /2

**Problem 4:** The time taken by a randomly selected applicant for filling out a form for Capilano University student loan has a normal distribution with mean value 10 minutes and standard deviation 2 minutes. If five students fill out a form on one day and six on another, what is the probability that the sample average amount of time taken on each day is at most 11 minutes?

Score: /4

**Problem 5:** Given a random sample  $X_1, X_2, \dots, X_n$  from a symmetric distribution with mean  $\mu$  and standard deviation  $\sigma$ , circle all the unbiased estimators of  $\mu$ .

- trimmed mean  $\bar{x}_{\text{tr}(10)}$ ;   • midrange;   • sample mean  $\bar{x}$ ;   • mode,   and
- median  $\tilde{x}$ .

Of the unbiased estimators you circled, which is the best estimator? Why?

Score: /4

**Problem 6:** The joint and marginal pmf's of  $X$  and  $Y$  are partly given in the following table.

$X \setminus Y$	1	2	3	Total
1	$\frac{1}{6}$	0		$\frac{1}{3}$
2		$\frac{1}{4}$		$\frac{1}{3}$
3			$\frac{1}{4}$	
Total	$\frac{1}{6}$	$\frac{1}{3}$		1

a. Complete the table with exact values.

Score: /2

b. Find  $E(Y)$ .

Score: /2

c. Find  $\text{Cov}(X, Y)$ .

Score: /2

d. Are  $X$  and  $Y$  independent?

Score: /2

**Problem 7:** A sample of 26 offshore oil workers took part in a simulated escape exercise, resulting in the accompanying data on time (seconds) to complete the escape:

389	356	359	363	375	424	325	394	402	373	373	370	364
366	364	325	339	393	392	369	374	359	356	403	334	397

Assume the random variable  $X$  for escape time measured in seconds is normally distributed. Answer the following.

- a. Compute sample mean and sample standard deviation.

Score: /2

- b. Calculate an upper confidence bound for population mean escape time using a confidence level of 95%.

Score: /3

**Problem 8:** The following is a summary of data analysis on the girth, in centimetres, of grizzly bears for a test of  $H_0 : \mu = 100$  cm versus  $H_a : \mu \neq 100$  cm:

One-Sample Z: Girth  
 Test of  $\mu = 100$  vs  $\mu \neq 100$

$N$	Variable	Mean	Standard Deviation	Z	P
61	Girth	93.39	21.79	-2.37	0.018

- a. What is the conclusion if you test with  $\alpha = 0.02$ ?

Score: /1

- b. Identify the type of mistake you could have made in part a).

Score: /1

- c. Before data was actually collected, what was the probability of making the mistake in part a)?

Score: /1

- d. Suppose the rejection region is  $(-\infty, 92) \cup (108, \infty)$  when in fact the true mean girth is  $\mu = 90$  cm. Find the probability of a Type II error.

Score: /2

**Problem 9:** A sample of 50 lenses used in eyeglasses yields a sample mean thickness of 3.05 mm and a sample standard deviation of 0.34 mm. The desired true average thickness of such lenses is 3.20 mm. Does the data strongly suggest that the true average thickness of such lenses is something other than what is desired? Test using  $\alpha = 0.05$ .

a. State null hypothesis.

Score: /1

b. State alternative hypothesis.

Score: /1

c. State test statistic and compute.

Score: /2

d. State conclusion.

Score: /1