Math 205 Spring 2016 Dr. Lily Yen	# Midterm 1 Show all your work	Name: ____________________ Score: ___/56

Throughout the midterm, use 4-significant-place accuracy unless specified.

Problem 1: *Aortic stenosis* refers to a narrowing of the aortic valve in the heart. The paper "Correlation Analysis of Stenotic Aortic Valve Flow Patterns Using Phase Contrast MRI" from Annals of Biomedical Engineering, 2005: 878-887 gave the following data on aortic root diameter in centimetres and gender for a sample of patients having various degrees of aortic stenosis:

M:	3.7	3.4	3.7	4.0	3.9	3.8	3.4	3.6	3.1	4.0	3.4	3.8	3.5
F:	3.8	2.6	3.2	3.0	4.3	3.5	3.1	3.1	3.2	3.0			

Compare and contrast the diameter observations for the two genders regarding location of the centre and the spread.

For males, the mean is $\bar{x}_{\mathrm{M}} = 3.64\,\mathrm{cm}$ while the standard deviation is $s_{\mathrm{M}} = 0.26\,\mathrm{cm}$. For females the corresponding numbers are $\bar{x}_{\mathrm{F}} = 3.28\,\mathrm{cm}$ and $s_{\mathrm{F}} = 0.45\,\mathrm{cm}$.
Clearly the aortic root diameter is larger for men but with smaller spread.

Score: /5

Problem 2: On Halloween, Mrs. Hyde gave out candy bars, protein bars, and other treats. Let C denote the event "the next treat given out is a candy bar," and T, the event "the next treat given out is a protein bar." Suppose $P(C) = 0.6$, and $P(T) = 0.3$. Give exact answers.

a. Find percentage of treats which are neither candy bars nor protein bars. **10%**

Assuming that a protein bar cannot at the same time be a candy bar, the chance of "other treats" is $1 - (P(C) + P(T)) = 0.1$.

b. Find $P(C \cup T)$. **90%**

Again, assuming that $C \cap T = \emptyset$, $P(C \cup T) = P(C) + P(T) = 0.9$.

c. Find $P(C' \cap T')$. **10%**

$C' \cap T'$ means "not C and not T", so "neither C nor T," which is the question asked in Part a.

d. Find $P(C \cup T')$. **70%**

$C \cup T' = (C \cap T) \cup T' = \emptyset \cup T' = T'$, so $P(C \cup T') = P(T') = 1 - 0.3 = 0.7$

e. Find $P(C \mid T')$. **6/7**

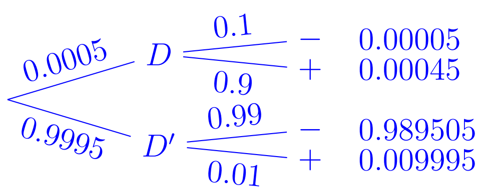
Since $C = (C \cap T) \cup (C \cap T') = \emptyset \cup (C \cap T') = C \cap T'$, it follows that $P(C \mid T') = \frac{P(C \cap T')}{P(T')} = \frac{P(C)}{P(T')} = \frac{0.6}{1-0.3} = 6/7$

Score: /5

Problem 3: A screening test for prostate cancer looks at the level of PSA (prostate-specific antigen) in the blood. There are a number of reasons besides prostate cancer that a man can have elevated PSA levels. In addition, many types of prostate cancer develop so slowly that they are never a problem. Unfortunately, there is currently no test to distinguish the different types and using the test is controversial because it is hard to quantify the accuracy rates and the harm done by false positives.

For this problem, we call a positive test a true positive if it catches a dangerous type of prostate cancer. We assume the following numbers: • The rate of prostate cancer among men over 50 is 0.0005; • True positive rate for the test is 0.9, and • False positive rate for the test is 0.01. Let $-$ be the event a man has a negative result from the test, and let D denote the event that a man has a dangerous type of the disease.

a. Draw the probability tree for the disease, then test results. Use the tree to find the probability of a negative test result.



$P(-) = 0.00005 + 0.989505 = 0.989555$

Score: /4

b. Given that a man tested negative, find the probability that he has the dangerous type of the disease.

$P(D \mid -) = \frac{P(D \cap -)}{P(-)} = \frac{0.00005}{0.989555} = 0.000\,050\,527\,8$

Score: /2

Problem 4: Consider a box of distinct text books, 5 statistics, 3 discrete mathematics, 2 linear algebra. Leave your answers in binomial coefficients or factorials.

a. How many ways can you line up 6 of these books on a shelf? **151200**

$\frac{10!}{4!} = 151200$

b. How many ways can you choose 4 books from the box? **210** Score: /2

$\frac{10!}{4!\,6!} = 210$

c. Choose 3 books at random from the box, find the probability that you get one of each subject area.

You can choose one in each subject in $\frac{5\times3\times2}{3!} = 5$ ways (without order). You can choose three book (in any subject(s)) in $\frac{10!}{3!\,7!} = 120$ ways. Hence the probability is $\frac{5}{120} = \frac{1}{24}$.

Score: /2

d. Choose 2 books at random from the box, find the probability that both books are from the same subject area.

You can choose two statistics books in $\frac{5!}{2!\,3!} = 10$ ways, two discrete math books in 3 ways, and two linear algebra books in 1 way. Hence you can choose two book in the same subject in $10 + 3 + 1 = 14$ ways.
You can choose any two books in $\frac{10!}{2!\,8} = 45$ ways, so the probability of two books in the same subject is $\frac{14}{45}$.

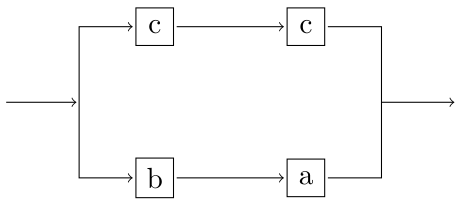
Score: /2

e. The books are selected one at a time at random without replacement, find the probability that the second book is a statistics text.

Since you know nothing about the first book chosen, this must equal the probability that the first book is a stats book, so $\frac{5}{10} = \frac{1}{2}$.

Score: /2

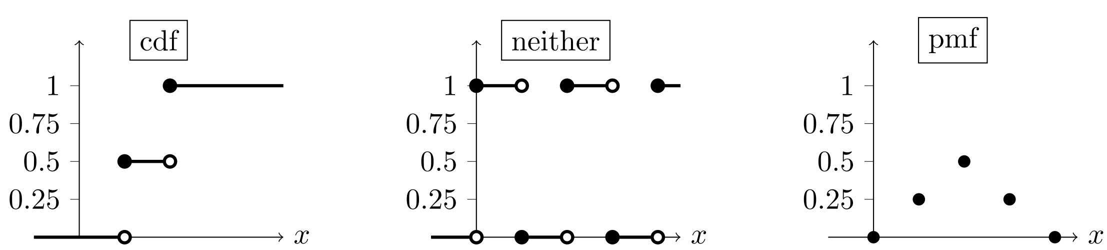
Problem 5: Consider the system of 3 types of components connected as shown. If the components work independently of one another, and let X denote the event that component x works where $X = \{A, B, C\}$ and $x = \{a, b, c\}$. Given $P(A) = 0.92$, $P(B) = 0.95$, $P(C) = 0.98$, compute the reliability of the system.



The upper part works if both c's work, which happens with probability $(0.98)^2 = 0.9604$. The lower part works if both the a and the b works, which has probability $0.92 \times 0.95 = 0.8740$. The chance that the system fails is therefore $(1 - 0.9604)(1 - 0.8740) = 0.0049896$, so the chance that it works is 0.9950104.

Score: /4

Problem 6: For each of the following graphs, determine whether it can be the graph of a cdf or a pmf or neither. Give reasons for each.



A cdf is increasing from 0 to 1. For a pmf the sum function values are between 0 and 1 and add to 1.

Score: /6

Problem 7: For each of the following, a random variable is defined from an experiment consisting of a sequence of independent trials. Name the distribution and specify all relevant parameters.

a. Keep tossing a fair coin until a HEAD appears. Let X be the number of tosses.

geometric distribution

The probability of a HEAD is $\alpha = \frac{1}{2}$. The pmf of X is $p(x) = (1-\alpha)^{x-1}\alpha$.

b. Keep tossing a fair coin until three HEADs appear. Let Y be the number of TAILS preceding the third HEAD.

negative binomial distribution

The probability of success, that is HEAD, is $p = \frac{1}{2}$, and the pmf is $\mathrm{nb}(y; 3, p)$.

c. From a bag of coins consisting 10 nickels and 20 dimes, randomly draw 5 coins without replacement where each set of 5 coins is equally likely to be chosen. Define Z to be the number of dimes.

hypergeometric distribution

The population size is $N = 30$, the sample size is $n = 5$, the number of successes is $M = 20$, and the pmf is $\mathrm{h}(z; 5, 20, 30)$.

d. Suppose 1 in 200 people carries the defective gene causing cancer. In a sample of 1000 individuals, define W to be the number who carry this gene.

Poisson distribution

The probability is $p = \frac{1}{200}$ and the sample size is $n = 1000$. Since $np = 5 < 10$, you may approximate the binomial distribution with a Poisson distribution, $\mathrm{p}(x; 5)$.

Score: /8

Problem 8: CapTen tennis racket comes in a midsize version and an oversize version. Sixty percent of all customers at Cap sports store want the oversize version. You must state the function and relevant parameters used in your graphing calculator.

a. Among 10 randomly selected customers who want CapTen rackets, find the probability that at least 6 want the oversize version.

$1 - \mathrm{binomialcdf}(10, 0.6, 5) = 0.6331$

b. Among 10 randomly selected customers, what is the probability that the number who want the oversize version is within 1 standard deviation of the mean value?

Since $n = 10$ and $p = 0.6$, $\mu = np = 6$ and $\sigma = \sqrt{np(1-p)} = 1.549$. Hence $P(\mu - \sigma < X < \mu + \sigma) = P(4.451 < X < 7.549) =$ $\mathrm{binomialpdf}(10, 0.6, 5) + \mathrm{binomialpdf}(10, 0.6, 6) + \mathrm{binomialpdf}(10, 0.6, 7) = 0.6665$

Score: /4

Problem 9: A trial has just resulted in a hung jury because 8 members of the jury were in favour of a guilty verdict and the other 4 for acquittal. If the jurors leave the jury room in random order and each of the first four leaving the room is accosted by a reporter in quest of an interview,

a. what is the probability mass function of X, the number of jurors favouring acquittal among those interviewed?

$$p(x) = \begin{cases} \mathrm{h}(0;4,4,12) = 0.1414 & x = 0 \\ \mathrm{h}(1;4,4,12) = 0.4525 & x = 1 \\ \mathrm{h}(2;4,4,12) = 0.3394 & x = 2 \\ \mathrm{h}(3;4,4,12) = 0.0646 & x = 3 \\ \mathrm{h}(4;4,4,12) = 0.0020 & x = 4 \end{cases}$$

Score: /4

b. How many of those favouring acquittal do you expect to be interviewed?

$E(p(x)) = n \cdot \frac{M}{N} = 4 \cdot \frac{4}{12} = \frac{4}{3} \approx 1.3$

Score: /2

Problem 10: Grasshoppers are distributed at random in Cap's sports field according to a Poisson process with parameters $\alpha = 2$ per square metre. How large should the radius R of a circular sampling region be taken so that the probability of finding at least one in the region equals 0.99?

If $P(X \geq 1) = 0.99$, then $P(X = 0) = 0.01$. In a Poisson distribution, $P(X = x) = e^{-\mu}\frac{\mu^x}{x!}$ where $\mu = 2a$ and a is the area of the circle.
Thus $0.01 = P(X = 0) = e^{-2a}$, so $2a = \ln(0.01)$, so $a = 2.302\,585\,093\,\mathrm{m}^2$. Since the area of a circle is $a = \pi R^2$, it follows that $R = 0.856\,116\,579\,9\,\mathrm{m}$.

Score: /4