Probability : {A Supplement}
(A "Light" Treatment?)

P.2 Fundamentals

Some basic definitions:
- An *experiment* is any procedure which produces an outcome that cannot be predicted with certainty in advance.
- An *event* is any collection of outcomes from an experiment.
- A *simple event* is a special type of event that cannot be broken down into simpler outcome pieces.
- A *sample space* for an experiment is the complete collection of all simple events.
  The letter S is commonly used to designate a sample space.

Example

Experiment:	Roll a fair die **once**.
Event:	A = {face showing up is even} = {2, 4, 6}
Simple Event:	E = {Face showing up is a 4} = {4}
Sample Space:	S = {1, 2, 3, 4, 5, 6}

Notation for Probability
The probability of event A is denoted by **P(A)**

Relative Frequency Principle
If an experiment is repeated over and over, then the probability of any event A

- is *approximated by* the relative frequency of A:

Relative frequency of event A

$$P(A) \approx \frac{\text{the number of times that A occurred}}{\text{the number of times that the experiment was repeated}}$$

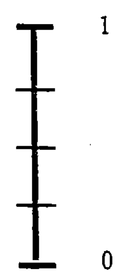
- is *equal to* the limiting long run relative frequency of A as the experiment is repeated forever.

Equally Likely Principle
If the simple events of an experiment are *equally likely*, then the probability of any event A

- is *equal to* $P(A) = \dfrac{\text{the number of simple events in A}}{\text{the number of simple events in the experiment}}$

Basic Axioms of Probability

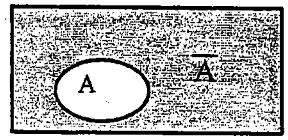
1. All probabilities are numbers between 0 and 1 inclusive.
2. The probability of an impossible event is 0.
3. The probability of an event certain to happen is 1.



Probability Scale

Complement Event

For any event A the complement of A is the event that consists of all simple events in the sample space that are not in the event A. The complement of A is denoted by $\overline{A}$.



Examples

1. Suppose **one** card is drawn from a deck of 52 cards. What is the probability that it is

   a) a Club? P(C) = 13/52 = 1/4

   b) an Ace? P(A) = 4/52 = 1/13

   c) a Red card? P(R) = 26/52 = 1/2

   d) a Red King? P(Red King) = 2/52 = 1/26

   e) not an Ace? $P(\overline{A})$ = 48/52 = 12/13

   f) not a Diamond? $P(\overline{D})$ = 39/52 = 3/4

	Club	**Diamond**	**Heart**	**Spade**
K	*	*	*	*
Q	*	*	*	*
J	*	*	*	*
10	*	*	*	*
9	*	*	*	*
8	*	*	*	*
7	*	*	*	*
6	*	*	*	*
5	*	*	*	*
4	*	*	*	*
3	*	*	*	*
2	*	*	*	*
A	*	*	*	*
	Black	**Red**	**Red**	**Black**

2. Roll a fair die **twice**. Find the probability of event

   a) A= the sum of the two faces is 7.

   P(A) = 6/36 = 1/6

   b) B = the product of the two faces is at least 15

   P(B) = 13/36

		Second Roll					
		1	**2**	**3**	**4**	**5**	**6**
First Roll	**1**	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
	2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
	3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
	4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
	5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
	6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Sample Space

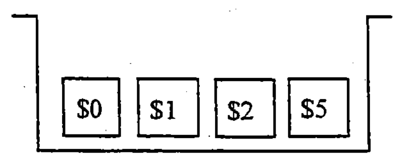
3. Randomly select **two** bills from the pot (without replacement). What is the probability that

   a) the **sum** of the two bills is greater than \$2?

   P(sum is greater than \$2) = 8/12 = 2/3

   b) the **product** of the two bills is not \$0?

   P(product is not \$0) = 6/12 = 1/2



		Second Bill			
		0	**1**	**2**	**5**
First Bill	**0**	—	(0,1)	(0,2)	(0,5)
	1	(1,0)	—	(1,2)	(1,5)
	2	(2,0)	(2,1)	—	(2,5)
	5	(5,0)	(5,1)	(5,2)	—

Sample Space

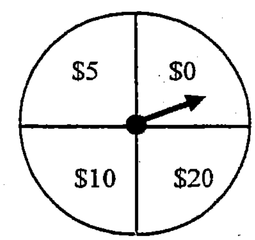
Problems For You To Do (Section P.2)

1. In a random sample of college students, 120 smoked cigarettes and 630 did not. From this sample estimate the probability that a college student smokes.

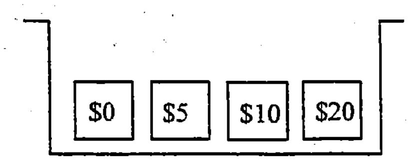
2. One card is drawn from a well-shuffled deck of 52 cards. Use the graphical display of the sample space to find the probability of getting
   a) a red card
   b) a jack
   c) a card that is not a jack
   d) a red card and a jack (a red jack)
   e) a red card or a jack

	Club	Diamond	Heart	Spade
K	*	*	*	*
Q	*	*	*	*
J	*	*	*	*
10	*	*	*	*
9	*	*	*	*
8	*	*	*	*
7	*	*	*	*
6	*	*	*	*
5	*	*	*	*
4	*	*	*	*
3	*	*	*	*
2	*	*	*	*
A	*	*	*	*

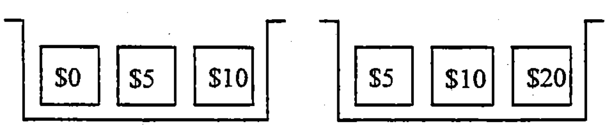
3. The pointer is spun twice.
   a) Use a table to construct a sample space of 16 equally likely simple events.
   b) Find the probability that
      i) you win a total of $10.
      ii) the same number comes up on each spin.



4. Two bills are selected from the pot without replacement.
   a) Use a table to construct a sample space of 12 equally likely simple events.
   b) Find the probability that
      i) you win a total of $10.
      ii) the second bill selected is the $20.



5. One bill is randomly selected from Pot A and one from Pot B.
   a) Use a table to construct a sample space.
      Use a tree diagram to construct a sample space.
      Which is easier?

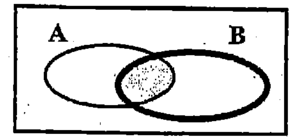


Pot A Pot B

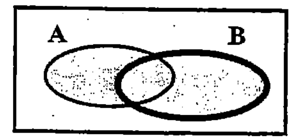
   b) What is the probability that the sum of the two bills is $10 ?

P.3 The Addition Rule

A and B
The event that both A and B occur is denoted by **A and B**

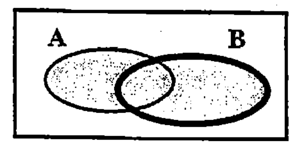


A or B
The event that A or B occurs (or both) is denoted by **A or B**



Addition Rule

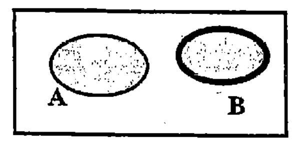
a) General Case: **P(A or B) = P(A) + P(B) – P(A and B)**



b) Special Sub-case: If A and B are **mutually exclusive** then

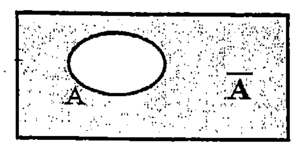
P(A or B) = P(A) + P(B)

No simple events in common



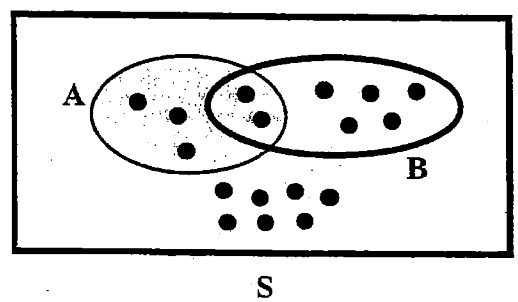
Probability of the Complement of A

P($\overline{A}$) = 1 – P(A)



Examples

1. If **one** of the 17 equally likely outcomes in the sample space S is randomly selected, find the probability that



a) **A** occurs. P(**A**) = 5/17

b) **A** does not occur. P($\overline{\mathbf{A}}$) = 12/17

c) both **A** and **B** occur. P(**A** and **B**) = 2/17

d) **A** but not **B** occurs. P(**A** and $\overline{\mathbf{B}}$) = 3/17

e) **B** but not **A** occurs. P($\overline{\mathbf{A}}$ and **B**) = 5/17

f) neither **A** nor **B** occurs. P($\overline{\mathbf{A}}$ and $\overline{\mathbf{B}}$) = 7/17

g) **A** or **B** occur.
This can be done by direct count P(**A** or **B**) = 10/17
or
it can be done using the Addition Rule: P(**A** or **B**) = 5/17 + 7/17 – 2/17 = 10/17

2. Randomly select **one** card from a deck of 52 playing cards.

   What is the probability that the card is

   a) a Queen or a Heart?

   P(Q or H) = 4/52 + 13/52 – 1/52 = 16/52

   b) an Ace and a Diamond ?

   P(A and D) = 1/52

	Club	**Diamond**	**Heart**	**Spade**
K	*	*	*	*
Q	*	*	*	*
J	*	*	*	*
10	*	*	*	*
9	*	*	*	*
8	*	*	*	*
7	*	*	*	*
6	*	*	*	*
5	*	*	*	*
4	*	*	*	*
3	*	*	*	*
2	*	*	*	*
A	*	*	*	*

3. The blood group and Rh type for 47 people in a study is summarized in the table to the right.

	A	**B**	**AB**	**O**	
Rh+	10	7	5	13	**35**
Rh-	3	2	1	6	**12**
	13	**9**	**6**	**19**	**47**

   If **one** person is selected randomly from the 47 what is the probability that the person

   a) is in blood group **A** and is type **Rh+**? P(A and Rh+) = 10/47

   b) is in blood group **A**? P(A) = 13/47

   c) is type **Rh+** ? P(Rh+) = 35/47

   d) is in blood group **A** or is type **Rh+**? P(A or Rh+) = 38/47 (direct count)

   or

   P(A or Rh+) = 13/47 +35/47 – 10/47 = 38/47

4. There are 31 students in a class. Sixteen students surf the Internet and 11 students use e-mail. Of these students, 6 students do both. What is the probability that a randomly selected student in the class surfs the Internet or uses e-mail?

   Solution a) Using the Addition Rule:

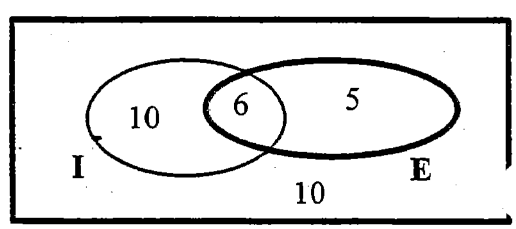
   I = Internet
   P(I) = 16/31
   P(I and E) = 6/31

   E = e-mail
   P(E) = 11/31

   P(I or E) = P(I) + P(E) – P(I and E) = 16/31 + 11/31 – 6/31 = 21/31

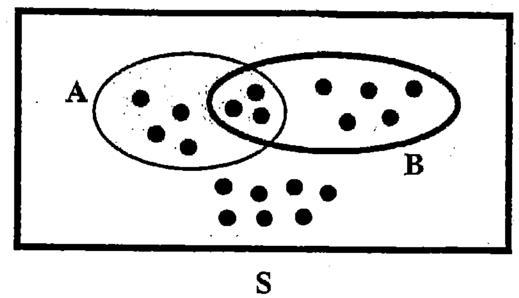
   Solution b) Using a Venn diagram:

   P(I or E) = 21/31



Problems For You To Do (Section P.3)

1. If **one** of the 19 equally likely outcomes in the sample space S is randomly selected, find the probability that
   a) **A** occurs.
   b) **A** does not occur.
   c) both **A** and **B** occur.
   d) **A** but not **B** occurs.
   e) **B** but not **A** occurs.
   f) neither **A** nor **B** occurs.
   g) **A** or **B** occur.
   h) at least one of **A** or **B** occurs.
   i) at most one of **A** or **B** occurs.



2. If one card is dealt from a deck of 52, what is the probability that it is
   a) a red card?
   b) a red card and a king?
   c) a red card or a king?
   d) a jack or a king?
   e) an ace or a diamond?

3. Thirty leg joint injuries were cross-classified according to the location of the injury and the treatment.

	Knee	Ankle	Hip	
Physio	4	5	2	11
Surgery	9	2	8	19
	13	7	10	30

   If one injury is selected randomly from the 30 what is the probability that the injury was
   a) a knee injury treated by surgery?
   b) not an ankle injury?
   c) treated by physio but not a hip injury?
   d) a hip injury or an injury treated by physio?
   e) a knee or an ankle injury?

4. Wilma the web whiz submits bids on two web design projects. She thinks she has a 70% chance of getting the first project, but just a 50% chance of getting the second. She puts only a 25% chance on getting neither one of the two projects. Find the probability that she gets
   a) both projects.
   b) at least one of the two projects.
   c) only the first project.
   d) only one of the two projects.

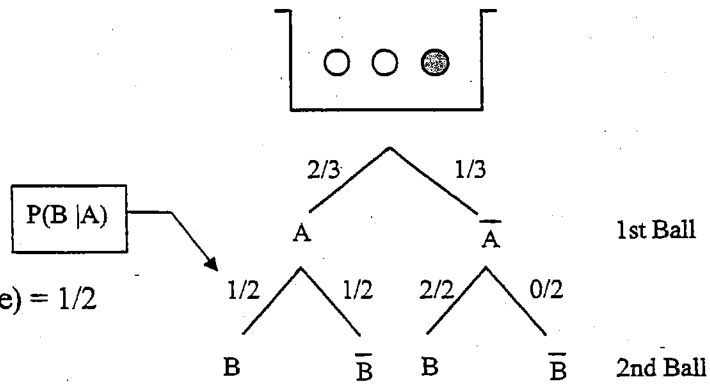
P.4 The Multiplication Rule

Conditional Probability
The probability of event B happening, *knowing that that event A has happened,* is called the **conditional probability of B given A**, and is denoted by **P(B|A)**.

Example 1 Drawing two balls from the pot, *without* replacement

A ball is randomly selected from the pot and **is not** replaced. Then a second ball is drawn.

Define the following events:
A: The first ball is white
B: The second ball is white



Find
$P(A) = 2/3$

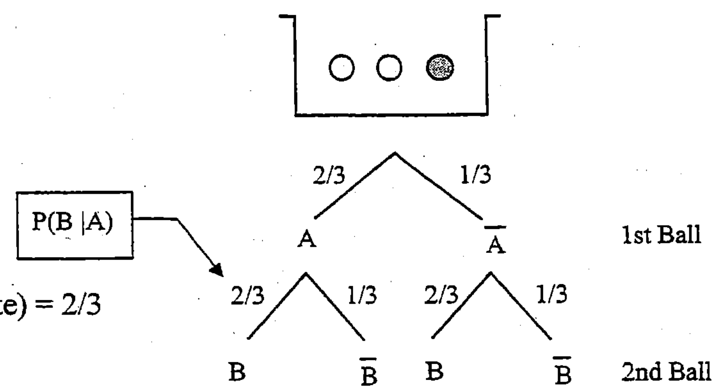
P(B *given* that the first ball drawn is white) = 1/2
$P(B \mid A) = 1/2$

$P(A \text{ and } B) = P(A) \cdot P(B \mid A) = 2/3 \cdot 1/2 = 1/3$

Example 2 Drawing two balls from the pot, *with* replacement

A ball is randomly selected from the pot and **is** replaced. Then a second ball is drawn.

Define the following events :
A: The first ball is white
B: The second ball is white



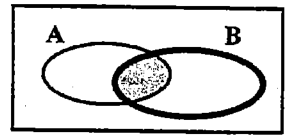
Find
$P(A) = 2/3$

P(B *given* that the first ball drawn is white) = 2/3
$P(B \mid A) = 2/3$

$P(A \text{ and } B) = P(A) \cdot P(B \mid A) = 2/3 \cdot 2/3 = 4/9$

Multiplication Rule

a) General Case: $\mathbf{P(A \text{ and } B) = P(A) \cdot P(B|A)}$



Equivalently: $\mathbf{P(B|A) = \dfrac{P(A \text{ and } B)}{P(A)}}$

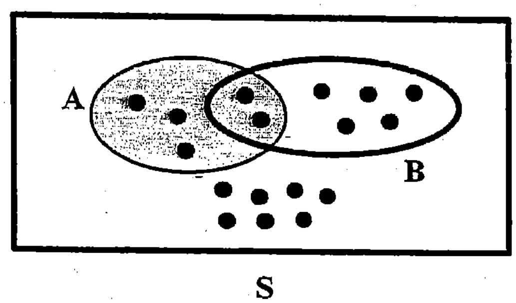
b) Special Sub-case: Events A and B are **independent** if $\mathbf{P(B|A) = P(B)}$

For independent events: $\mathbf{P(A \text{ and } B) = P(A) \cdot P(B)}$

Two events that are not independent are called **dependent**.

Examples

1. a) If **one** of the 17 equally likely outcomes in the sample space S is randomly selected, find



i) $P(B|A) = 2/5$

ii) $P(A \text{ and } B) = 2/17$ (direct count)

or

$P(A \text{ and } B) = 5/17 \cdot 2/5 = 2/17$ (Multiplication Rule)

iii) $P(A|B) = 2/7$

b) Are A and B independent or dependent?

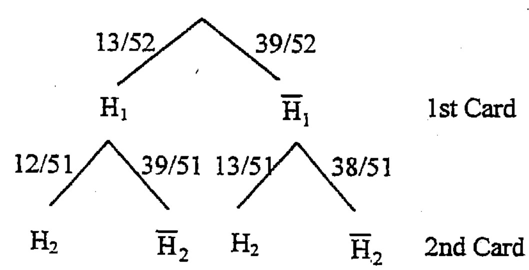
$P(B) = 7/17$

$P(B|A) = 2/5$

Since P(B) is NOT equal to P(B|A) we conclude that events A and B are **dependent**.

2. Two cards are randomly dealt from a deck of 52 cards (without replacement). What is the probability that

a) both cards are hearts?



$P(H_1 \text{ and } H_2) = P(H_1) \cdot P(H_2 \mid H_1)$

$= 13/52 \cdot 12/51 = 156/2652 = 1/17$

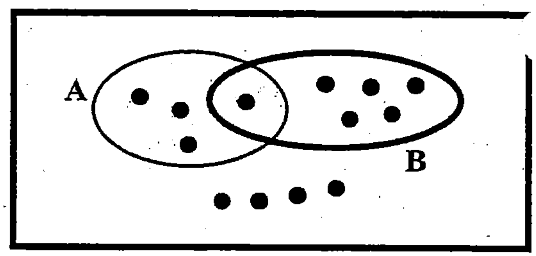
b) only one of the two cards is a heart?

$P((H_1 \text{ and } \overline{H}_2) \text{ or } (\overline{H}_1 \text{ and } H_2)) = P(H_1 \text{ and } \overline{H}_2) + P(\overline{H}_1 \text{ and } H_2)$

$13/52 \cdot 39/51 + 39/52 \cdot 13/51 = 1014/2652 = 13/34$

Problems For You To Do (Section P.4)

1. If the 13 simple events are equally likely find



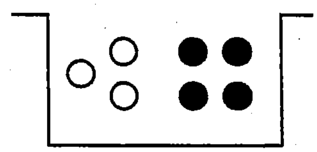
   a) $P(\overline{A} \text{ or } B) =$

   b) $P(A \text{ and } \overline{B}) =$

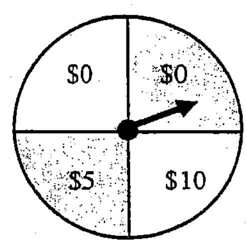
   c) $P(B|A) =$ d) $P(A|\overline{B}) =$

   e) $P(\overline{A \text{ and } B}) =$ f) $P(\overline{A \text{ or } B}) =$

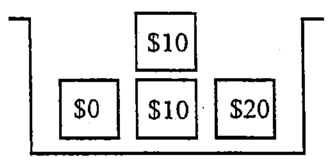
2. Randomly select **two** balls from the pot (without replacement). Find the probability that
   a) both balls selected are white.
   b) exactly one ball is white.
   c) at least one ball is white.



3. Spin the pointer **twice**. Find the probability that
   a) Only one spin ends up in a shaded region.
   b) The sum of the two spins is $10.



4. Randomly select **two** bills from the pot



   a) ***without*** replacement.
   Find the probability that
   i) the sum of the two bills is $20.
   ii) the $0 bill is the second bill selected.

   b) ***with*** replacement.
   Find the probability that
   i) the sum of the two bills is $20
   ii) the $0 bill is the second bill selected.

5. Randomly deal **two** cards from a deck of 52 (without replacement). Find the probability that
   a) both cards are clubs.
   b) both cards are kings.

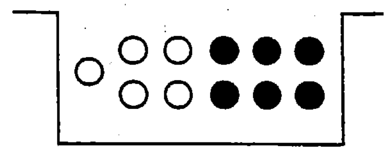
6. A fair die is rolled **twice**. Find the probability that
   a) each roll comes up a 6.
   b) neither roll comes up a 5.

7. Lisa and Ian work independently on the same problem. If Lisa has a 90% chance of solving the problem and Ian has a 70% chance of solving it, what is the probability that
   a) both solve the problem?
   b) at least one of them solves the problem?

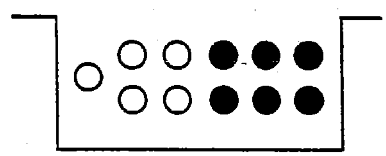
8. A survey of smokers found that of 1000 women, 200 were heavy smokers and 20 had emphysema. Of those who had emphysema, 13 were heavy smokers. Are being heavy smokers and having emphysema independent?

9. A web page designer bids on two contracts. She estimates that she has a 40% chance of getting the first contract and a 50% chance of getting the second contract. If the contracts are decided independently, what is the probability that she gets **exactly one** of the two contracts?

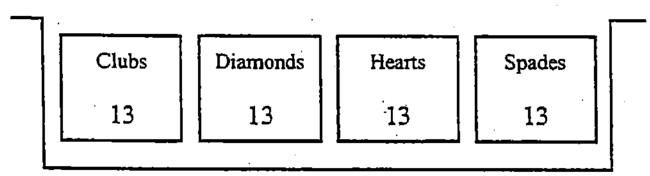
10. Randomly select **three** balls from the pot (without replacement). Find the probability that you get three black balls.



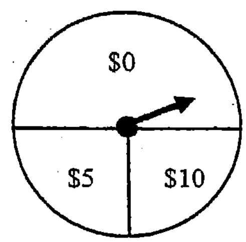
11. Randomly select **five** balls from the pot (without replacement). Find the probability that you get
   a) five black balls.
   b) at least 1 white ball.



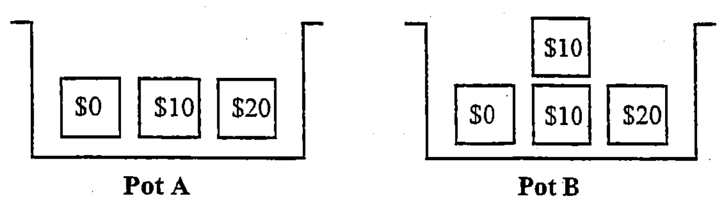
12. Randomly deal **five** cards from a deck of 52. Find the probability that you get
   a) five clubs.
   b) at least one club.



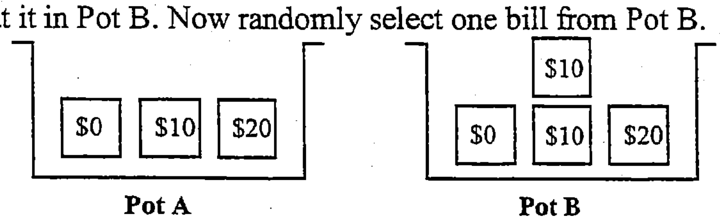
13. Spin the pointer **ten** times. Find the probability that you get
   a) no $0's.
   b) at least one $0.



14. Randomly select one bill from Pot A and one from Pot B. What is the probability you get a total of $20 for the two bills?



15. Randomly select one bill from Pot A and put it in Pot B. Now randomly select one bill from Pot B. What is the probability that you select a $10 bill from Pot B?



16. For the experiment in Question 15, if the bill selected from Pot B was a \$10 bill, what is the probability that a \$20 bill was transferred from Pot A to B?

17. The BC Lottery Corporation advertises that there is a 1/28 chance that you will win *some* prize each time that you play the BC_{49} game. If you play this game once a week for the next 52 weeks, what is the chance that you win *some* prize at least once?

18. The blood group and Rh type for 47 people in a study is summarized in the table to the right.

	A	B	AB	O	
Rh+	10	7	5	13	**35**
Rh-	3	2	1	6	**12**
	13	**9**	**6**	**19**	**47**

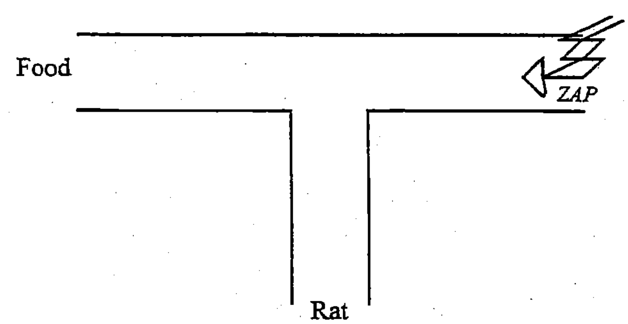
a) If **one** person is selected randomly from the 47 what is the probability that the person
   i) is type **Rh+** but is not in blood group **B**?
   ii) is in blood group **A** or is **Rh-**?

b) What is the probability that a person who is type **Rh-** is from blood group **O**?

c) If **two** people are selected randomly from the 47 without replacement, what is the probability that
   i) they have different Rh types?
   ii) they have the same blood type?

d) If **five** people are selected randomly from the 47 without replacement, what is the probability that at least one has blood type A?

19. In a T-maze, a rat is given food if she turns left and an electric shock (ouch!) if she turns right. On the first run there is a 50:50 chance that the rat will turn either way. Then, if she received food on the first run, the probability that she turns left on the second run is 0.68. However, if she received a shock on the first run, the probability of turning left on the second run is 0.84.



a) What is the probability that the rat receives food on both runs?
b) What is the probability that the rat receives food on the second run?
c) If a rat turns left on the second run what is the probability that it had also turned left on the first?

20. In a particular social services program
    70% are single parents
    60% are employed
    80% of all the single parents are employed.

a) When one person is randomly selected from this program what is the chance that he/she is a single parent or employed?
b) What percentage of the employed people are single parents?

21. The table below shows 18 possible appointment times in a dentist's appointment schedule.

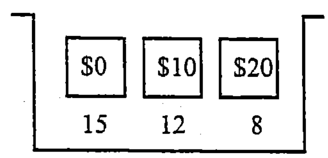
	Monday	Tuesday	Wednesday	Thursday
Morning	____	____	____ ____	____ ____
Afternoon	____ ____ ____ ____	____ ____ ____	____ ____ ____ ____	____

a) If Erica randomly selects **one** appointment time from the 18,
   i) what is the probability that she picks a morning time?
   ii) what is the probability that she picks an afternoon time or a time on Tuesday?
   iii) for which day of the week are the events "she picks morning" (M) and "she picks [day]" (D) **independent**. Support your answer with a probability argument.

One of Monday, Tuesday, Wednesday, or Thursday

b) If Thomas randomly selects one appointment time from the 17 remaining after Erica picks hers, what is the probability that at least one of them picks Wednesday?

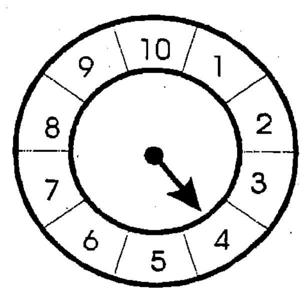
22. Randomly select three bills from the pot (without replacement).



a) What is the probability that you get three $0 bills?

b) What is the probability that you get one bill of each type?

23. Spin the pointer three times. What is the probability that



a) all three spins come up at 7?

b) each of the three spins comes up at a different number?

Sec. P2 Problems:

1. $P(\text{smokes}) = \dfrac{120}{120+630} = \dfrac{120}{750} = \dfrac{4}{25} = 0.16$

2. (a) $P(\text{Red}) = \dfrac{26}{52} = \dfrac{1}{2}$ (b) $P(\text{Jack}) = \dfrac{4}{52} = \dfrac{1}{13}$

(c) $P(\overline{\text{Jack}}) = \dfrac{48}{52} = \dfrac{12}{13}$ (d) $P(\text{Red Jack}) = \dfrac{2}{52} = \dfrac{1}{26} \approx 0.0385$

(e) $P(R \text{ or } J) = \dfrac{28}{52} = \dfrac{7}{13} \approx 0.5385$

3. (a)

		Second Spin			
		0	5	10	20
First Spin	0	(0,0)	(0,5)	(0,10)	(0,20)
	5	(5,0)	(5,5)	(5,10)	(5,20)
	10	(10,0)	(10,5)	(10,10)	(10,20)
	20	(20,0)	(20,5)	(20,10)	(20,20)

(b) (i) $P(T=10) = \dfrac{3}{16} = 0.1875$

(ii) $P(\text{same}) = \dfrac{4}{16} = \dfrac{1}{4} = 0.25$

(a)

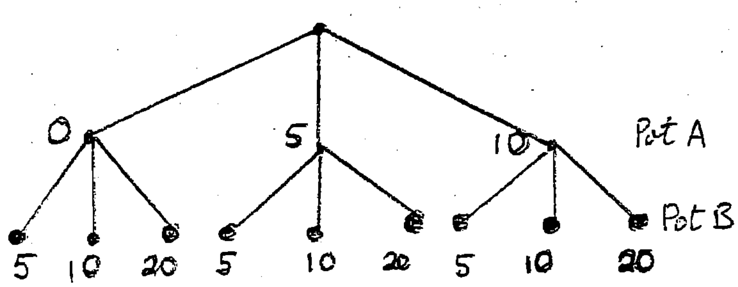
		Second Bill			
		0	5	10	20
First Bill	0	—	(0,5)	(0,10)	(0,20)
	5	(5,0)	—	(5,10)	(5,20)
	10	(10,0)	(10,5)	—	(10,20)
	20	(20,0)	(20,5)	(20,15)	—

(b) (i) $P(T=10) = \dfrac{2}{12} = \dfrac{1}{6}$

(ii) $P(\text{Second}=20) = \dfrac{3}{12} = \dfrac{1}{4} = 0.25$

5. (a)

		Pot B		
		5	10	20
Pot A	0	(0,5)	(0,10)	(0,20)
	5	(5,5)	(5,10)	(5,20)
	10	(10,5)	(10,10)	(10,20)



(b) $P(\text{Sum}=10) = 2/9$

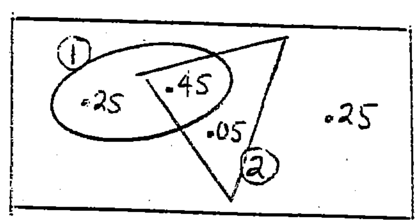
Sec. P.3 Problems:

1. (a) $P(A) = 7/19$ (b) $P(\overline{A}) = 12/19$
   (c) $P(A \text{ and } B) = 3/19$ (d) $P(A \text{ and } \overline{B}) = 4/19$
   (e) $P(B \text{ and } \overline{A}) = 5/19$ (f) $P(\overline{A} \text{ and } \overline{B}) = 7/19$
   (g) $P(A \text{ or } B) = 12/19$ (h) $P(A \text{ or } B) = 12/19$
   (i) $P(\overline{A \text{ and } B}) = 1 - P(A \text{ and } B) = 1 - 3/19 = 16/19$

2. (a) $P(\text{Red}) = 26/52 = 1/2$
   (b) $P(\text{Red and King}) = 2/52 = 1/26$
   (c) $P(\text{Red or King}) = 28/52 = 7/13$
   (d) $P(\text{Jack or King}) = 8/52 = 2/13$
   (e) $P(\text{Ace or Diamond}) = 16/52 = 4/13$

3. (a) $P(K \text{ and } S) = 9/30 = 3/10$
   (b) $P(\overline{A}) = 23/30$
   (c) $P(P \text{ and } \overline{H}) = 9/30 = 3/10$
   (d) $P(H \text{ or } P) = 19/30$
   (e) $P(K \text{ or } A) = 20/30 = 2/3$

4. $P(\overline{①} \text{ and } \overline{②}) = 0.25$



$\therefore P(① \text{ or } ②) = 1 - 0.25$
$= 0.75$

$P(① \text{ or } ②) = P(①) + P(②) - P(① \text{ and } ②)$

(a) $P(① \text{ and } ②) = P(①) + P(②) - P(① \text{ or } ②) = .70 + .50 - .75 = 0.45$

(b) $P(① \text{ or } ②) = 0.75$

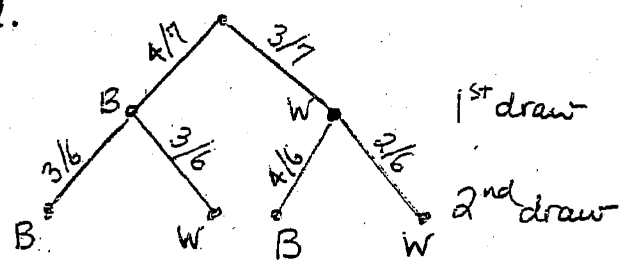
(c) $P(① \text{ and } \overline{②}) = 0.25$

(d) $P([① \text{ and } \overline{②}] \text{ or } [② \text{ and } \overline{①}]) = .25 + .05 = 0.30$

Sec. P.4 Problems:

1(a) 10/13 (b) 3/13 (c) 1/4 (d) 3/7 (e) 12/13 (f) 4/13

2.



(a) $P(W_1 \text{ and } W_2) = \frac{3}{7}\cdot\frac{2}{6} = \frac{6}{42} = \frac{1}{7}$

(b) $P((B_1 \text{ and } W_2) \text{ or } (W_1 \text{ and } B_2))$

$= \frac{4}{7}\cdot\frac{3}{6} + \frac{3}{7}\cdot\frac{4}{6} = \frac{24}{42} = \frac{4}{7}$

(c) $1 - P(B_1 \text{ and } B_2) = 1 - \frac{4}{7}\cdot\frac{3}{6} = \frac{30}{42} = \frac{5}{7}$

3. let $W = \{\text{pointer ends in white region}\}$; $S = \{\text{pointer ends in shaded region}\}$

(a) $P((S_1 \text{ and } W_2) \text{ or } (W_1 \text{ and } S_2)) = \frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{2}$

(b) $P((0 \text{ and } 10) \text{ or } (10 \text{ and } 0) \text{ or } (5 \text{ and } 5)) = \frac{1}{2}\cdot\frac{1}{4} + \frac{1}{4}\cdot\frac{1}{2} + \frac{1}{4}\cdot\frac{1}{4} = \frac{5}{16}$

4.(a) (i) $P((0 \text{ and } 20) \text{ or } (20 \text{ and } 0) \text{ or } (10 \text{ and } 10)) = \frac{1}{4}\cdot\frac{1}{3} + \frac{1}{4}\cdot\frac{1}{3} + \frac{2}{4}\cdot\frac{1}{3} = \frac{1}{3}$

(ii) $P(\overline{0} \text{ and } 0) = \frac{3}{4}\cdot\frac{1}{3} = \frac{1}{4}$

(b)(i) $P((0 \text{ and } 20) \text{ or } (20 \text{ and } 0) \text{ or } (10 \text{ and } 10)) = \frac{1}{4}\cdot\frac{1}{4} + \frac{1}{4}\cdot\frac{1}{4} + \frac{2}{4}\cdot\frac{2}{4} = \frac{3}{8}$

(ii) $P((\overline{0} \text{ and } 0) \text{ or } (0 \text{ and } 0)) = \frac{3}{4}\cdot\frac{1}{4} + \frac{1}{4}\cdot\frac{1}{4} = \frac{1}{4}$

5.(a) $P(C_1 \text{ and } C_2) = \frac{13}{52}\cdot\frac{12}{51} = \frac{1}{17}$

(b) $P(K_1 \text{ and } K_2) = \frac{4}{52}\cdot\frac{3}{51} = \frac{1}{221}$

6.(a) $P(6 \text{ and } 6) = \frac{1}{6}\cdot\frac{1}{6} = \frac{1}{36}$

(b) $P(\overline{5} \text{ and } \overline{5}) = \frac{5}{6}\cdot\frac{5}{6} = \frac{25}{36}$

7. let $L = \{\text{Lisa solves problem}\}$; $I = \{\text{Ian solves problem}\}$

(a) $P(L \text{ and } I) = P(L)\cdot P(I)$ (independence!)

$= (0.90)(0.70) = 0.63$

(b) $P(L \text{ or } I) = P(L) + P(I) - P(L \text{ and } I)$

$= 0.90 + 0.70 - 0.63 = 0.97$

8. let $H = \{\text{heavy smoker}\}$; $E = \{\text{having emphysema}\}$;

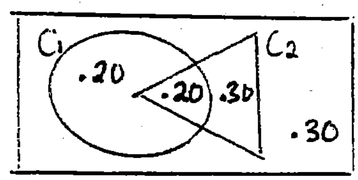
$P(H) = \frac{200}{1000} = \frac{1}{5}$; $P(H \mid E) = \frac{13}{20}$;

since $P(H) \neq P(H \mid E)$, the events "being heavy smoker" and "having emphysema" are dependent.

9. let $C_1 = \{\text{getting first contract}\}$; $C_2 = \{\text{getting second contract}\}$

$P(C_1 \text{ and } C_2) = P(C_1) \cdot P(C_2)$ (independence)

$= (0.40)(0.50) = 0.20$



$P((C_1 \text{ and } \overline{C_2}) \text{ or } (\overline{C_1} \text{ and } C_2)) = 0.20 + 0.30$

$= 0.50$

10. $P(B_1 \text{ and } B_2 \text{ and } B_3) = \frac{6}{11} \cdot \frac{5}{10} \cdot \frac{4}{9} = \frac{4}{33} \approx 0.1212$

11. (a) $P(B_1 \text{ and } B_2 \text{ and } B_3 \text{ and } B_4 \text{ and } B_5) = \frac{6}{11} \cdot \frac{5}{10} \cdot \frac{4}{9} \cdot \frac{3}{8} \cdot \frac{2}{7} = \frac{1}{77} \approx 0.0130$

(b) $P(\text{at least one white}) = 1 - P(\text{all black}) = 1 - \frac{1}{77} = \frac{76}{77} \approx 0.9870$

12. (a) $P(C_1 \text{ and } C_2 \text{ and } C_3 \text{ and } C_4 \text{ and } C_5) = \frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} \cdot \frac{10}{49} \cdot \frac{9}{48} \approx 0.0004952$

(b) $1 - P(\text{no clubs}) = 1 - P(\overline{C_1} \text{ and } \overline{C_2} \text{ and } \overline{C_3} \text{ and } \overline{C_4} \text{ and } \overline{C_5})$

$= 1 - \frac{39}{52} \cdot \frac{38}{51} \cdot \frac{37}{50} \cdot \frac{36}{49} \cdot \frac{35}{48} \approx 0.7785$

13. (a) $P(\overline{O}_1 \text{ and } \overline{O}_2 \text{ and} \ldots \text{ and } \overline{O}_{10}) = P(\overline{O}_1) \cdot P(\overline{O}_2) \cdot \ldots \cdot P(\overline{O}_{10}) = \frac{1}{2} \cdot \frac{1}{2} \cdot \cdots \cdot \frac{1}{2} = \frac{1}{2^{10}}$

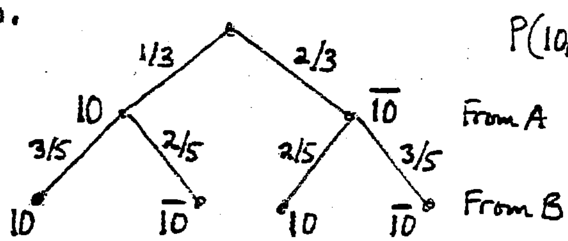
$= \frac{1}{1024} \approx 0.0009766$

(b) $1 - P(\text{no } 10\text{'s}) = 1 - \frac{1}{1024} = \frac{1023}{1024} \approx 0.9990$

14. $P((0_A \text{ and } 20_B) \text{ or } (20_A \text{ and } 0_B) \text{ or } (10_A \text{ and } 10_B))$

$= \frac{1}{3} \cdot \frac{1}{4} + \frac{1}{3} \cdot \frac{1}{4} + \frac{1}{3} \cdot \frac{2}{4} = \frac{1}{3}$

15.



$P(10_B) = P((10_A \text{ and } 10_B) \text{ or } (\overline{10}_A \text{ and } 10_B))$

$= \frac{1}{3} \cdot \frac{3}{5} + \frac{2}{3} \cdot \frac{2}{5} = \frac{7}{15}$

16. $P(20_A \mid 10_B) = \dfrac{P(20_A \text{ and } 10_B)}{P(10_B)} = \dfrac{\frac{1}{3}\cdot\frac{2}{5}}{\frac{7}{15}} = \dfrac{2}{7}$

17. $1 - P(\text{no prize every week}) = 1 - \left(\frac{27}{28}\right)^{52} \approx 0.8491$

18. (a) (i) $P(Rh+ \text{ and } \bar{B}) = 28/47$

(ii) $P(A \text{ or } Rh-) = 22/47$

(b) $P(O \mid Rh-) = 6/12 = 1/2$

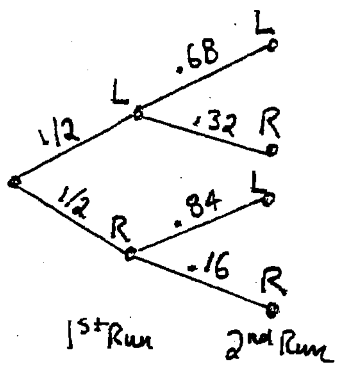
(c) (i) $P((Rh+ \text{ and } Rh-) \text{ or } (Rh- \text{ and } Rh+)) = \frac{35}{47}\cdot\frac{12}{46} + \frac{12}{47}\cdot\frac{35}{46} \approx 0.3885$

(ii) $P((A \text{ and } A) \text{ or } (B \text{ and } B) \text{ or } (AB \text{ and } AB) \text{ or } (O \text{ and } O))$

$= \frac{13}{47}\cdot\frac{12}{46} + \frac{9}{47}\cdot\frac{8}{46} + \frac{6}{47}\cdot\frac{5}{46} + \frac{19}{47}\cdot\frac{18}{46} \approx 0.2775$

(d) $1 - P(\text{none has A}) = 1 - \frac{34}{47}\cdot\frac{33}{46}\cdot\frac{32}{45}\cdot\frac{31}{44}\cdot\frac{30}{43} \approx 0.8186$

19.



(a) $P(L_1 \text{ and } L_2) = \frac{1}{2}(.68) = 0.34$

(b) $P(L_2) = P((L_1 \text{ and } L_2) \text{ or } (R_1 \text{ and } L_2))$

$= \frac{1}{2}(.68) + \frac{1}{2}(.84) = 0.76$

(c) $P(L_1 \mid L_2) = \dfrac{P(L_1 \text{ and } L_2)}{P(L_2)} = \dfrac{0.34}{0.76} \approx 0.4474$

20. let $S = \{\text{single parent}\}$; $E = \{\text{employed}\}$

(a) $P(S \text{ or } E) = P(S) + P(E) - P(S \text{ and } E)$

$= 0.70 + 0.60 - P(S \text{ and } E)$

But $P(S \text{ and } E) = P(S)\cdot P(E \mid S) = (0.70)(0.80) = 0.56$

$\therefore P(S \text{ or } E) = 0.70 + 0.60 - 0.56 = 0.74$

(b) $P(S \mid E) = \dfrac{P(S \text{ and } E)}{P(E)} = \dfrac{0.56}{0.60} \approx 0.9333$

$\therefore$ 93.3% of employed people are single parents.

21. (a) (i) P (morning) = 6/18 = 1/3

(ii) P (afternoon or Tues) = 13/18

(iii) Wednesday, because $P(\text{Wed.}) = \frac{6}{18} = \frac{1}{3}$

and $P(\text{Wed} \mid \text{morning}) = \frac{2}{6} = \frac{1}{3} = P(\text{Wed.})$

∴ "picking morning" and "picking Wed." are independent.

(b) $1 - P(\text{neither picks Wed.}) = 1 - \frac{12}{18} \cdot \frac{11}{17} = \frac{29}{51} \approx 0.5686$

22. (a) $P(0 \text{ and } 0 \text{ and } 0) = \frac{15}{35} \cdot \frac{14}{34} \cdot \frac{13}{33} = \frac{13}{187} \approx 0.0695$

(b) P((0 and 10 and 20) or (0 and 20 and 10) or (10 and 0 and 20) or (10 and 20 and 0) or (20 and 0 and 10) or (20 and 10 and 0))

$$= \frac{15 \cdot 12 \cdot 8 + 15 \cdot 8 \cdot 12 + 12 \cdot 15 \cdot 8 + 12 \cdot 8 \cdot 15 + 8 \cdot 15 \cdot 12 + 8 \cdot 12 \cdot 15}{1440 \cdot 1439 \cdot 1438}$$

≈ 0.2200

23. (a) $P(7 \text{ and } 7 \text{ and } 7) = \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} = 0.001$

(b) P((1 and a different no. and another diff. no.) or (2 and a diff. no. and another diff. no.) or ...)

$= \frac{1}{10} \cdot \frac{9}{10} \cdot \frac{8}{10} + \frac{1}{10} \cdot \frac{9}{10} \cdot \frac{8}{10} + \ldots$ (10 terms)

$= (.072)(10) = 0.72$

Or

$= \frac{10}{10} \cdot \frac{9}{10} \cdot \frac{8}{10} = \frac{720}{1000} = 0.72$

first spin any number

second spin any number except that which occurred on first spin

third spin any number except the two that occurred on first & second spins