

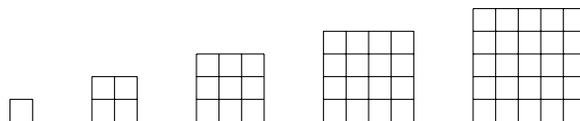
# Test 1

Show all your work

Name: \_\_\_\_\_

Score: \_\_\_/50

**Problem 1:** Consider the pattern of square numbers below:



Count the number of squares in each grid, and write the number of squares below each grid. You have then the first five square numbers.

a. What is the tenth square number?

100

b. What is the  $n$ -th square number?

$n^2$

c. What type of integer is added to go from one square number to the next one?

odd

Score: /3

**Problem 2:** Explain the connection between the following two problems.

a. Find the number of intersections among 6 lines in general position.

15

b. Find the number of handshakes among 6 people who all shake hands with one another.

15

Do you get the same answer for both problems?  Yes How many handshakes will there be among 20 people?

190

The number is 15 for both because each handshake corresponds to an intersection between two lines.

Among 20 people, we need  $1 + 2 + \dots + 19 = \frac{20 \times 19}{2} = 190$  handshakes.

Score: /4

**Problem 3:** Jean-Paul has some coins in his pocket totalling 55 cents. He knows that he only has nickels and dimes, at least one of each. List all possible scenarios in Jean-Paul's pocket.

A nickel is 5 cents and a dime is 10 cents.

$55 = 1 \times 10 + 9 \times 5 = 2 \times 10 + 7 \times 5 = 3 \times 10 + 5 \times 5 = 4 \times 10 + 3 \times 5 = 5 \times 10 + 1 \times 5$ , so five possibilities.

Score: /4

**Problem 4:** Sarah had five times as many stickers as Maddie did, and twice as many as April. April had 25 stickers. How many stickers did they have altogether?

Say Sarah had  $S$  stickers, Maddie  $M$ , and April  $A$ . Then  $S = 5M = 2A$  and  $A = 25$ .

Therefore  $S = 50$  and  $M = 10$ .

Together they then have  $S + M + A = 50 + 10 + 25 = 85$  stickers.

Score: /3

**Problem 5:** Justine read three novels, with the first having 120 more pages than the second, and the second having 75 fewer pages than the third. Order the novels from the least number of pages to the most.

Say the three novels have  $x$ ,  $y$ , and  $z$  pages, respectively. Then  $x = y + 120$  and  $y = z - 75$ . Thus  $x = (z - 75) + 120 = z + 45$ . Clearly  $z - 75 < z < z + 45$ , so  $y < z < x$ . Thus the 2nd book has fewer pages than the 3rd, which in turn has fewer than the first.

Score: /4

**Problem 6:** Convert the following numbers to Hindu Arabic numerals. Show all steps.

a. MCMLXXIV



b. 

c. 

a. 1974 (Roman)

b. 257 (Mayan)

c. 379 (Babylonian)

Score: /6

**Problem 7:** When you count in base 4, what are the next six numbers following 1, 2, 3, 10, 11, 12, 13, 20, 21, 22, 23?

30, 31, 32, 33, 100, 101

Score: /3

**Problem 8:** Which is larger?  $143_6$  or  $81_9$ ?

$81_9 = 8 \cdot 9^1 + 1 \cdot 9^0 = 72 + 1 = 73$  is larger than  
 $143_6 = 1 \cdot 6^2 + 4 \cdot 6^1 + 3 \cdot 6^0 = 36 + 24 + 3 = 63$ :

$$81_9 < 143_6$$

Score: /2

**Problem 9:** Subtract  $223_5$  from  $2431_5$ . Indicate clearly the base of your answer.

$$2203_5 = 303_{10}$$

Score: /3

**Problem 10:** Let  $S = \{1, 3, 5, x, y, z\}$ ,  $A = \{1, 2, 3, 4, 5\}$ , and  $B = \{2, 3, 4, 5, y\}$ .

a. Find  $A \cup B =$   $\{1, 2, 3, 4, 5, y\}$ .

b. Is  $(A \cap B) \subset S$ ?

Since  $2 \in A \cap B = \{2, 3, 4, 5\}$  but  $2 \notin S$ ,  $(A \cap B) \not\subset S$ .

Score: /3

**Problem 11:** Out of 28 students in Math 190, 27 students own laptop computers and 26 own cellphones. No student owns neither. How many own both? Draw a Venn diagram to represent the problem.

Let  $L$  be the set of students who own cellphones, and let  $M$  be the set that own computers. Then  $|M| = 27$ ,  $|L| = 26$ , and  $|U| = 28$ . Since  $\overline{M \cap L} = \overline{M} \cup \overline{L} = \emptyset$ ,  $|M| + |L| - |M \cap L| = |U|$ , so  $27 + 26 - |M \cap L| = 28$ , so  $|M \cap L| = 27 + 26 - 28 = 25$ , so 25 students own both computer and cellphone.

Score: /3

**Problem 12:** For each example from children's ways of reasoning, if it is wrong, explain the mistake and correct it. If it is correct, explain the child's logic.

a. 
$$\begin{array}{r} 55 \\ + 48 \\ \hline 913 \end{array}$$
 The 1 in 13 should be a carry in the tens' column

b. 
$$\begin{array}{r} 36 \\ \times 8 \\ \hline 2448 \end{array}$$
 The 4 in 48 should be a carry in the tens' column

c.  $364 - 79$

$$\begin{array}{r} 364 \\ - 79 \\ \hline 300 \text{ Subtract just the hundreds} \\ - 10 \text{ Subtract tens: } 60 - 70 = -10 \\ \hline 290 \text{ } 300 - 10 = 290 \\ - 5 \text{ Subtract ones: } 4 - 9 = -5 \\ \hline 285 \text{ } 290 - 5 = 285, \text{ the correct answer} \end{array}$$

d.  $280 \div 35$

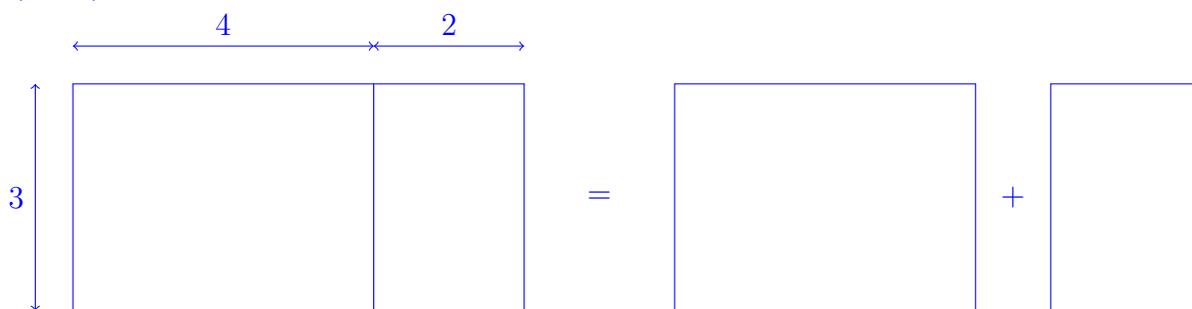
$$\begin{array}{r} 280 \text{ Repeatedly subtract pairs of 35s} \\ - 70 \\ \hline 210 \\ - 70 \\ \hline 140 \\ - 70 \\ \hline 70 \\ - 70 \\ \hline 0 \end{array}$$

So, four 70's is equal to eight 35's. The answer is 8.

Score: /8

**Problem 13:** Illustrate the distributive property of multiplication over addition using rectangular arrays (areas). First quote the property.

$$c(a + b) = ca + cb$$



Score: /4