Math 190
Spring 2012
Dr. Lily Yen

Test 2

Show all your work

Name: ____________________

Score: ___/45

Problem 1: Explain subtraction for $53 - 26$ in the given ways.

a. Method 1: Change both the minuend and the subtrahend to make it a simpler problem.

b. Method 2: Regroup subtrahend to make it an easier problem.

c. Method 3: Use missing-addend approach.

a. $(53 + 4) - (26 + 4) = 57 - 30 = 27$

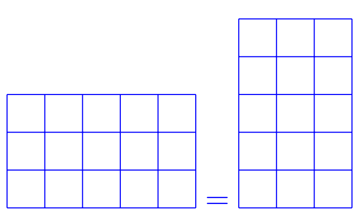
b. $53 - (23 + 3) = 53 - 23 - 3 = 30 - 3 = 27$

c. $26 + \boxed{?} = 53$, $26 + \boxed{24 + 3} = 50 + 3$, $\boxed{?} = 27$.

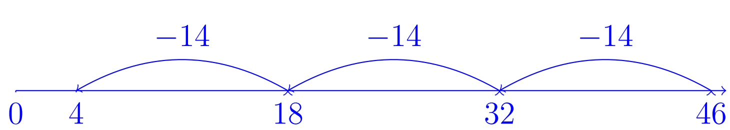
Score: /6

Problem 2: Illustrate the solution to each of the following using the given approach.

a. Use a rectangular array to show that $3 \times 5 = 5 \times 3$.



b. Use repeated subtraction on the number line to show the answer to $46 \div 14$ with remainder.



The quotient is 3 since we have three jumps of -14. The remainder is 4 since that's what's left after the three jumps.

Score: /4

Problem 3: What is the difference between *discrete* quantities and *continuous* quantities?

Discrete quantities can be counted, but continuous quantities must be measured.

Score: /2

Problem 4: Write a story problem to explain the Distributive Property of Multiplication over Addition.

First state what the property says.

$a \times (b + c) = a \times b + a \times c$

Suppose Eric buys b chocolate bars at \$$a$ each paying \$$a \times b$. He really likes the bars and goes back to buy c more bars at the same price. He has now paid a total of \$$a \times b +$ \$$a \times c$. If he had instead bought $b + c$ chocolate bars in the first place at \$$a$ each , he would have paid \$$a(b + c)$.

Score: /4

Problem 5: For the following questions, determine whether the statement is true or false. Give a reason for each answer.

a. The natural numbers are closed under subtraction.

Define the natural numbers first, then the closure property before answering the question.

False. The natural numbers are $\mathbb{N} = \{1, 2, 3, \dots\}$, and closure under subtraction means that if $a, b \in \mathbb{N}$, then $a - b \in \mathbb{N}$. However, $1, 3 \in \mathbb{N}$ but $1 - 3 = -2 \notin \mathbb{N}$.

b. Both addition and multiplication have all of the following three properties: closure, commutativity, and associativity over the integers. Part of your reasoning is to show what these properties are.

True. $\forall a, b, c \in \mathbb{Z}$, $a + b \in \mathbb{Z}$, $a + b = b + a$, $a + (b + c) = (a + b) + c)$, $a \times b \in \mathbb{Z}$, $a \times b = b \times a$, and $a \times (b \times c) = (a \times b) \times c$.

c. The identity element for addition is the same as that for multiplication.

False. Zero is the identity for addition: $0 + a = a + 0 = a$.
One is the identity for multiplication: $1 \times a = a \times 1 = a$.

Score: /6

Problem 6: Express the answer of each in scientific notation.

a. There are about 4×10^6 microbes on each square centimetre of your skin, and you have about 2×10^4 square centimetres of skin. How many microbes live on your skin?

$4 \times 10^6 \times 2 \times 10^4 = 8 \times 10^{10}$ microbes

b. In five days, 1 gram of fresh yeast can grow to about 750 000 grams enough to make about 50 000 loaves of bread. Find the number of grams of yeast needed to make one loaf of bread.

$750\,000 \div 50\,000 = 7.5 \times 10^5 \div (5 \times 10^4) = 1.5 \times 10^1 = 15$ grams of yeast per loaf.

c. Suppose Amy has 1.05×10^5 black hair; Bill has 9.8×10^3 brown hair; Christian has 8.7×10^4 red hair. If you combine Bill's hair with Christian's hair, would you get more than Amy's hair?

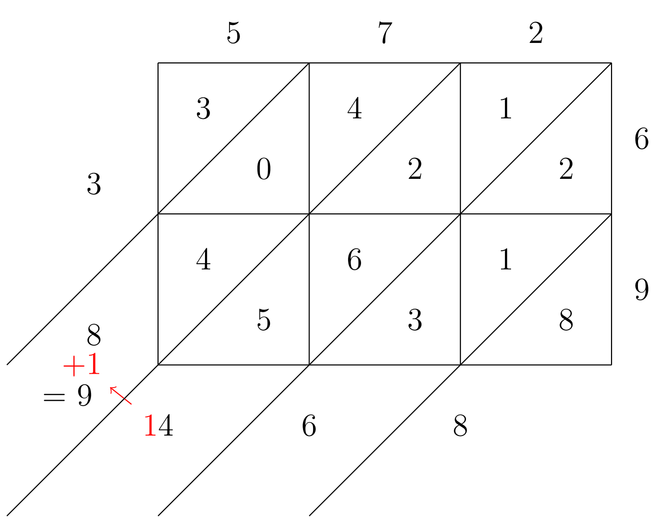
No. Bill and Christian together have
$9.8 \times 10^3 + 8.7 \times 10^4 = 0.98 \times 10^4 + 8.7 \times 10^4 = 9.68 \times 10^4$ hairs. Amy, on the other hand, has $1.05 \times 10^5 = 10.5 \times 10^4$ hairs, and $10.5 \times 10^4 > 9.68 \times 10^4$.

d. Compute $0.000\,000\,000\,024 \div 0.000\,000\,04$.

$0.000\,000\,000\,024 \div 0.000\,000\,04 = 24 \times 10^{-12} \div (4 \times 10^{-8}) = (24 \div 4) \times 10^{-12-(-8)} = 6 \times 10^{-4} = 0.000\,6$.

Score: /8

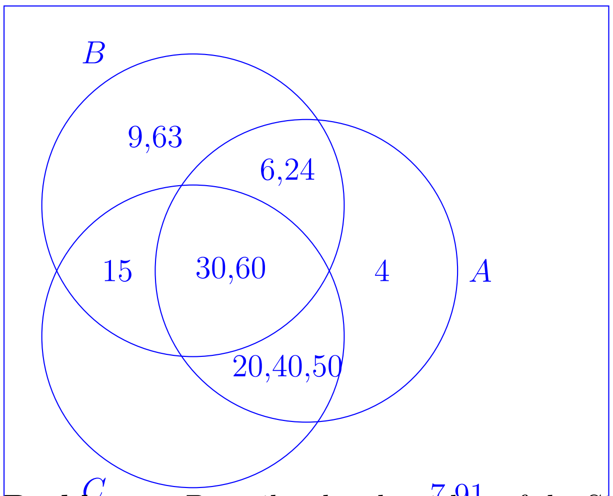
Problem 7: Use the lattice method for multiplication to demonstrate 572×69. Check your answer using the standard algorithm for multiplication.



$$\begin{array}{r} 572 \\ \times 69 \\ \hline 5148 \\ 3432 \\ \hline 39468 \end{array}$$

Score: /3

Problem 8: Draw a Venn diagram for the following set of numbers such that the universe contains three sets: Set A contains all even numbers, set B, all numbers divisible by 3, and set C, all those divisible by 5 in the universe. The universe $U = \{4, 6, 7, 9, 15, 20, 24, 30, 40, 50, 60, 63, 91\}$.



Score: /4

Problem 9: Describe the algorithm of the Sieve of Eratosthenes by finding the first 4 primes in the table below.

Table 1: The Sieve of Eratosthenes

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70

a. Cross a since it is a unit, neither prime, not composite.

b. The next number in the table is 2. Circle it and cross out every second number after 2.

c. The next number in the table is 3. Circle it and cross out every third number after 3.

d. The next number in the table is 5 (since 4 was crossed out in step **??**). Circle it and cross out every fifth number after 5.

e. The next number in the table is 7. Circle it and cross out every seventh number after 7. (Only 49 was not crossed out already.)

Score: /4

Problem 10: Find the GCF of 150 and 270.

30

Score: /2

Problem 11: What is the LCM of m and n where $m = 2^3 \times 5^4 \times 7^6 \times 11$ and $n = 3^2 \times 5^3 \times 11^4 \times 13$?

$2^3 \times 3^2 \times 5^4 \times 7^6 \times 11^4 \times 13$

Score: /2