

Math 126
Summer 2016
Dr. Lily Yen

Test 3

Show all your work

Name: _____

Score: ____/43

No Calculator permitted in this part. Read the questions carefully. Show all your work and clearly indicate your final answer. Use proper notation.

Problem 1: Use separation of variables to find the general solution of the following differential equation.

$$y' = y^2(1 - x^2)$$

Score: /4

Problem 2: Find all values of a such that $y = x^a$ is a solution of

$$y'' - 12x^{-2}y = 0$$

Score: /5

Problem 3: Find the first 5 terms of $a_n = a_{n-1} + 2a_{n-2}$ for $n > 2$ and $a_1 = 1$, $a_2 = 5$.

Score: /3

Problem 4: For each sequence or series below, determine if it converges; find the limit if possible, or prove that it diverges.

a. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{e^n}$ If this series converges, find its limit. Is it absolutely convergent?

Score: /3

b. $\sum_{n=2}^{\infty} \frac{1}{\ln(n)}$.

Score: /3

c. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n!}$.

Score: /3

Test 3

Show all your work

Name: _____

Calculators permitted in this part.

Problem 5: For the function $f(x) = \ln(\sin(x))$, set up the integral expression for its arc length for $x \in [\pi/4, \pi/2]$. Evaluate the integral accurate to 4 decimal places.

Score: /4

Problem 6: Find the orthogonal trajectories of $3x^2 + 2y^2 = c$, for arbitrary non-negative constants c . Sketch at least three members of each family (the function and its orthogonal trajectory). Label each curve with its parameter value.

Score: /5

Problem 7: A patient is given the drug theophylline intravenously at a rate of 43.2 mg/hour to relieve acute asthma. The rate at which the quantity of drug leaves the body (in mg/hour) is proportional to the quantity there, with proportionality constant 0.082 /hour. Assume the patient's body contains none of the drug initially.

a. Describe in words how you would expect the quantity of theophylline in the patient to vary with time and sketch an approximate graph.

b. Write a differential equation satisfied by the quantity of the drug, $A(t)$ in mg.

c. Solve the differential equation and graph the solution. What happens to the quantity in the long run?

Score: /8

Problem 8: Consider the infinite series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$$

- a. Evaluate the N th partial sum for the first four N 's.

Score: /2

- b. With your calculator, evaluate the 20-th partial sum. Give 4-decimal place accuracy.

Score: /2

- c. The sum is equal to $\pi^2/12$. Find the smallest N -th partial sum that is accurate to 4 decimal places of the answer.

Score: /1