Math 126
Summer 2007
Dr. Lily Yen

Test 3

Show all your work

Name: ____________________

Score: ___/30

No Calculator permitted in this part. Read the questions carefully. Show all your work and clearly indicate your final answer. Use proper notation.

Problem 1: For each of the following sequences, determine (with analytical reasons) whether the sequence converges or diverges. If the sequence converges, find its limit.

a. $\left\{(-1)^n \tan\left(\frac{(2n-1)\pi}{4}\right)\right\}_{n=0}^{\infty}$

b. $a_n = \frac{4^n}{(n+5)^{2n}}$

c. $\left\{\frac{\ln(n^2) - 2n^3}{4n^2+5}\right\}_{n=1}^{\infty}$

Score: /6

Problem 2: Evaluate

$$\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}+\frac{1}{56}+\frac{1}{72}+\frac{1}{90}= \boxed{}$$

Score: /2

Problem 3: Find the radius of convergence and the interval of convergence of each of the following power series.

a. $\displaystyle\sum_{n=0}^{\infty}\frac{2^n(x+1)^n}{(n+3)^2}$

b. $\displaystyle\sum_{n=1}^{\infty}\frac{(n+5)^2x^n}{\pi^n}$

c. $\displaystyle\sum_{n=0}^{\infty}n!(3x-7)^n$

Score: /6

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Problem 4: Consider the sequence $\{a_n\}_{n=1}^{\infty}$ defined recursively as follows:

$$a_1 = 2, \quad a_2 = 3, \quad a_n = \frac{a_{n-1} + a_{n-2}}{2}, \quad n \geq 3$$

Find a_{20} accurate to 3 decimal places. [ ]

Find $\lim_{n\to\infty} a_n$. [ ]

Score: /3

Problem 5: Let $f(x) = \ln(2 - x)$, find the 4th order Taylor Polynomial $P_4(x)$ for $f(x)$ about $a = 1$.

Determine graphically the largest interval containing $x = 1$ in which $P_4(x)$ approximates $f(x)$ to within 0.01. Give interval endpoints to 2 decimal places [ ], and list the Y-functions which are plotted on your TI.

$y_{\max} =$

$y_{\min} =$

$x_{\min} =$ $x_{\max} =$

Score: /5

Problem 6: Determine a power series representation for the function,

$$h(x) = \frac{x^2}{5 - x}$$

and determine the radius of convergence. Score: /3

Problem 7: Find the Taylor Series for the function $f(x) = 5^{-x}$ about $a = 2$. Give your final answer in simplified form using the $\sum$ notation. Also verify that the series converges to $f(x)$. You do not need to find the interval of convergence. Score: /5