

Math 126  
Summer 2007  
Dr. Lily Yen

## Test 2

Show all your work

Name: \_\_\_\_\_

Score: \_\_\_\_/46

**No Calculator permitted** in this part. Read the questions carefully. Show all your work and clearly indicate your final answer. Use proper notation.

**Problem 1:** Write a Riemann sum and then a definite integral representing the area of the region enclosed by the  $y$ -axis, the line  $3x + y = 6$ , and the parabola  $y = x^2 - 4$ . Find the area by evaluating the integral. Score: /3

**Problem 2:** Find the volume of the solid whose base is the region in the  $x$ - $y$ -plane bounded by the curves  $y = x^2$  and  $y = 8 - x^2$  and whose cross-sections perpendicular to the  $x$ -axis are squares with one side in the  $x$ - $y$ -plane. Score: /3

**Problem 3:** The region bounded by the curve  $y = e^{-x}$  and the  $x$ -axis between  $x = 0$  and  $x = 1$  is revolved around the  $x$ -axis. Find the volume of this solid of revolution.

Score: /3

**Problem 4:** Describe the solid whose volume is represented by the integral below.

$$\pi \int_0^1 (y^2 - y^4) dy$$

Score: /3

**Problem 5:** Find the volume of a wedge formed by a plane slicing a right-circular cylinder of radius  $r$  if the plane meets the base at an angle  $\theta$  and the line formed by its intersection with the base forms a diameter of the cylinder.

Score: /4

# Test 2

Show all your work

Name: \_\_\_\_\_

Calculators permitted from here on.

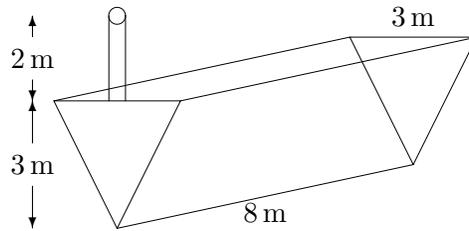
**Problem 6:** Find the length of the loop of the curve  $x(t) = 3t - t^3$  and  $y(t) = 3t^2$ .

Score: /4

**Problem 7:** If Lily's cup of tea has temperature  $95^\circ\text{C}$  in Cedar 136 where the temperature is  $22^\circ\text{C}$ , then, according to Newton's Law of Cooling, the temperature of the tea after  $t$  minutes is  $T(t) = 22 + 73e^{-t/50}$ . What is the average temperature of the tea during the first half hour?

Score: /4

**Problem 8:** The tank in the shape of a triangular prism is full of water.



a. Find the work required to pump the water out of the spout.

Score: /3

b. Suppose that the pump breaks down after  $4.7 \times 10^5$  J of work has been done. What is the depth of the water remaining in the tank?

Score: /3

**Problem 9:** The mouse population in Lily's neighbourhood is modelled by the differential equation

$$\frac{dP}{dt} = 1.2P \left( 1 - \frac{P}{4200} \right)$$

a. For what values of  $P$  is the population increasing?

b. For what values of  $P$  is the population decreasing?

c. What are the equilibrium solutions?

Score: /3

**Problem 10:** Find the orthogonal trajectories of the family of curves given by  $xy = k$ .

Score: /4

**Problem 11:** Solve the differential equation  $\cos(y)y' - \tan(x) = 0$  with initial condition  $y(0) = \pi/6$ .

Score: /4

**Problem 12:** The air in our classroom with volume  $400 \text{ m}^3$  initially contains  $0.15\%$  carbon dioxide. Fresher air with only  $0.05\%$  carbon dioxide flows into the room at a rate of  $2 \text{ m}^3/\text{min}$  and the mixed air flows out at the same rate. Find the percentage of carbon dioxide in the room as a function of time. What happens in the long run? Score: /5