Math 126
Spring 2017
Dr. Lily Yen

Test 3

Show all your work

Name: ____________________
Number: ____________________
Signature: ____________________
Score: ___/43

No Calculator permitted in this part. Read the questions carefully. Show all your work and clearly indicate your final answer. Use proper notation.

Problem 1: Match formulas with graphs for each sequence below where $n \geq 1$.

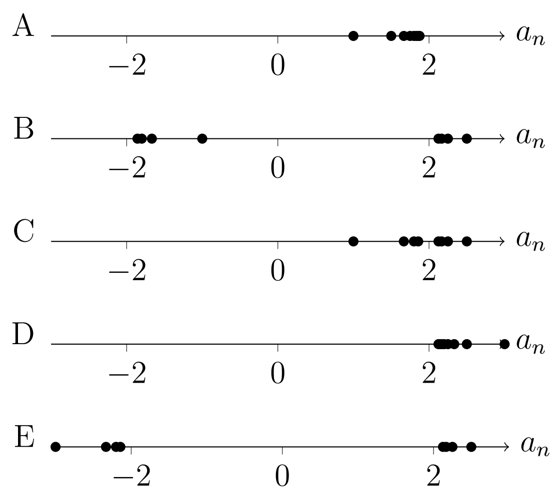
a. $a_n = 2 - 1/n$

b. $a_n = (-1)^n 2 + 1/n$

c. $a_n = 2 + 1/n$

d. $a_n = 2 + (-1)^n/n$

e. $a_n = (-1)^n 2 + (-1)^n/n$



Score: /5

Problem 2: The sequence a_n is defined recursively:

$$a_n = a_{n-1} + 4n - 3,$$

for $n > 1$, and $a_1 = 1$.

a. Compute a_2, a_3, a_4, and a_5.

b. Show that $a_n = 2n^2 - n$ satisfies the same recurrence relation.

Score: /4

Problem 3: Use the properties of geometric series to find the sum of the alternating series:

$$1 - \frac{z}{2} + \frac{z^2}{4} - \frac{z^3}{8} + \frac{z^4}{16} - \cdots$$

For what values of the variable z does the series converge to the sum you find?

Score: /3

Problem 4: For each series below, determine convergence or not with appropriate tests.

a. Let a be an arbitrarily fixed constant in the following series: $\displaystyle\sum_{n=1}^{\infty} \frac{n + a^n}{na^n}$.

Score: /2

b. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(3^n)}$

Score: /2

c. Let r be an arbitrarily fixed positive constant in the series: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{r^n n!}$. For which values of r is the series convergent?

Score: /3

Problem 5: Set up an integral expression for the arc length of the ellipse $x^2/4 + y^2/9 = 1$. DO NOT EVALUATE.

Score: /3

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Calculators permitted in this part.

Problem 6: First, graph the nth partial sum, S_n, in your graphing calculator, then estimate the error in approximating the sum by S_{99}.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+\sqrt{n}}$$

Score: /5

Problem 7: Recall Newton's law of cooling: the rate of change in temperature of an object varies proportionally to the difference in temperature between the object and the surrounding medium. Write a differential equation whose solution is the temperature as a function of time for a bottle of black currant juice taken out of a 5 °C refrigerator and left in a 20 °C kitchen, given that after one hour, the temperature of the bottle rose to 7 °C. Include all conditions. DO NOT SOLVE THE DE.

Score: /4

Problem 8: Find the orthogonal trajectories to the family of curves $xy = c$, for $c \neq 0$. Graph at least three representatives from each family on the same coordinate system.

Score: /5

Problem 9: When people smoke, carbon monoxide (CO) is released into the air. In a room of volume $60\,\mathrm{m}^3$ initially without CO, air containing $5\,\%$ CO is introduced at a rate of $0.002\,\mathrm{m}^3/\mathrm{minute}$. (This means that $5\,\%$ of the volume of the incoming air is CO.) The CO mixes immediately with the rest of the air and the mixture leaves the room at the same rate as it enters.

a. Write a differential equation for $y(t)$, the volume in cubic metres of CO at time t in minutes.

b. Solve the differential equation.

c. What happens to the value of $y(t)$ in the long run?

Score: /7