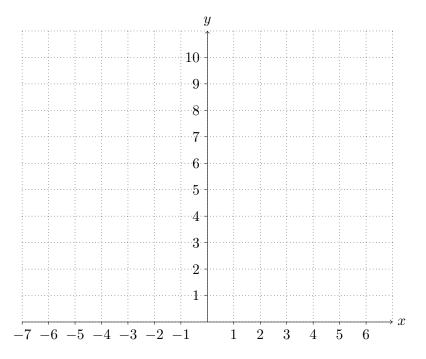
Math 126
Spring 2017
Dr. Lily Yen

## Test 2 Show all your work

Name:		
Number:		
Signature:		
Score:	/40	

No Calculator permitted in this part. Read the questions carefully. Show all your work and clearly indicate your final answer. Use proper notation.

**Problem 1**: Draw  $y = 2^x$  and  $x = \frac{3}{7}(y-1)(y-7)$  on the grid. One of the intersections of these two curves is (3,8). Shade the region bounded by these two curves.



Use integrals to express the following. DO NOT EVALUATE YOUR INTEGRALS. Draw a cross-sectional strip for each solid of rotation.

a. The area of the shaded region.

Score: /2

b. The volume of a solid that has the shaded region as its base, and cross-sections perpendicular to the y-axis are semi-circles.

Score: /2

c. The volume of the solid obtained by rotating the region around x = -6.

Score: /2

d. The volume of the solid obtained by rotating the region around y = 10.

Score: /2

**Problem 2**: Evaluate the following integrals analytically.

a. 
$$\int x^3 e^{x^2} dx$$

Score: /3

b. 
$$\int e^{-x} \cos(2x) \, dx$$

Score: /4

c. 
$$\int \frac{2x^2 - x - 1}{(x - 2)(x^2 + 1)} \, dx$$

Score: /5

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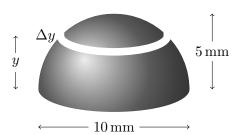
## Test 2

Show all your work

Name:	
Number	

Calculators permitted from here on.

**Problem 3**: Write a Riemann sum and then a definite integral representing the volume of the hemisphere, using the slice shown. Evaluate the integral exactly.



Score: /4

Problem 4: Evaluate if possible; otherwise, state why the integral does not exist.

$$\int_1^3 \frac{1}{(x-2)^2} \, dx$$

Score: /4 **Problem 5**: Suppose that f is continuous for all real numbers and that  $\int_0^\infty f(x) dx$  converges to a real number, c. Let a be any positive number. Decide which of the statements below are true and which are false. Give an explanation for your answer.

a. 
$$\int_0^\infty af(x) dx$$
 converges.

c. 
$$\int_0^\infty f(ax) dx$$
 converges.

b. 
$$\int_0^\infty f(x+a) dx$$
 converges.

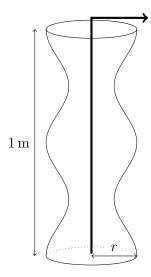
d. 
$$\int_0^\infty (a+f(x)) dx$$
 converges.

Score: /4

**Problem 6**: A 20 m uniform chain with a mass density of  $1.5 \,\mathrm{kg/m}$  is dangling from the roof of a building. How much work is required to pull half the chain up onto the top of the building? Use  $g = 9.81 \,\mathrm{m/s^2}$ . Provide your answer accurate to 2 decimal places.

Score: /4

**Problem 7**: A display fish tank has a circular cross section with radius r metres at a height of y metres above the ground given by  $r = (3 + \cos(4\pi y))/100$ . The height of the tank is 1 m. A spout is placed 0.05 m above the tank. Find the work required to pump all the water from a full tank out through the spout. Provide your answer accurate to the nearest integer. Hint:  $1 \,\mathrm{m}^3$  of water has a mass of  $1000 \,\mathrm{kg}$ . (Drawing is not to scale.)



Score: /4

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