Math 126
Spring 2017
Dr. Lily Yen

Test 1

Show all your work

Name: ______________

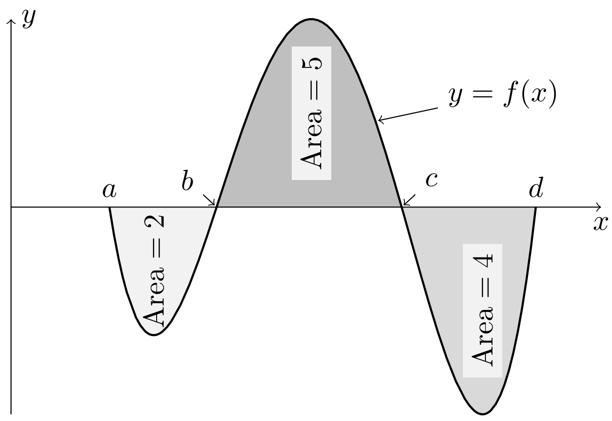
Number: ______________

Signature: ______________

Score: ___/45

No Calculator permitted in this part. Read the questions carefully. Show all your work and clearly indicate your final answer. Use proper notation.

Problem 1: Use the figure to find the values of the following.



a. $\int_a^b f(x)\,dx = \boxed{-2}$

b. $\int_a^c f(x)\,dx = \boxed{3}$

c. $\int_a^d |f(x)|\,dx = \boxed{11}$

Score: /3

Problem 2: Use the definition of the definite integral to write $\int_0^\pi e^{-x^2}\,dx$ as a limit of a Riemann sum. Specify your choice of summand as an R_n, L_n, or M_n.

$$\lim_{n\to\infty} L_n = \lim_{n\to\infty} \frac{\pi}{n}\sum_{j=0}^{n-1} e^{-(j\pi/n)^2} \quad \text{or} \quad \lim_{n\to\infty} R_n = \lim_{n\to\infty} \frac{\pi}{n}\sum_{j=1}^{n} e^{-(j\pi/n)^2}$$

$$\text{or} \quad \lim_{n\to\infty} M_n = \lim_{n\to\infty} \frac{\pi}{n}\sum_{j=1}^{n} e^{-\left(\frac{(2j-1)\pi}{2n}\right)^2}$$

Score: /2

Problem 3: Express the following in the form $\int_a^b f(t)\,dt$.

$$\lim_{n\to\infty}\left(\frac{1}{n}\sum_{j=1}^{n}\frac{\cos(2+\frac{j(5-\pi)}{n})}{\frac{j(5-\pi)}{n}}\right) = \boxed{\int_0^{5-\pi}\frac{\cos(2+x)}{(5-\pi)x}\,dx}$$

This is the right-hand Riemann sum for

$$\int_0^{5-\pi}\frac{\cos(2+x)}{(5-\pi)x}\,dx$$

Score: /4

$\boxed{\quad /9}$

Problem 4: Integrate the following analytically. Evaluate each definite integral.

a. $\displaystyle\int_{4a}^{2b} \cot(\theta/2)\csc(\theta/2)\,d\theta$

If $u = \theta/2$, then $\frac{du}{d\theta} = \frac{1}{2}$, so $d\theta = 2\,du$. Therefore $\int_{4a}^{2b} \cot(\theta/2)\csc(\theta/2)\,d\theta = \int_{\theta=4a}^{\theta=2b} 2\cot(u)\csc(u)\,du = -2\int_{2a}^{b} -\cot(u)\csc(u)\,du = -2\csc(u)\Big|_{2a}^{b} = -2\csc(b) + 2\csc(2a)$.

Score: /3

b. $\displaystyle\int \frac{e^{-x}}{(e^{-x}+5)^4}\,dx$

If $u = e^{-x} + 5$, then $\frac{du}{dx} = -e^{-x}$, so $dx = -\frac{1}{e^{-x}}$. Therefore

$$\int \frac{e^{-x}}{(e^{-x}+5)^4}\,dx = \int \frac{e^{-x}}{u^4}\frac{-1}{e^{-x}}\,du = -\int u^{-4}\,du$$
$$= \frac{1}{3}u^{-3} + C = \frac{1}{3(e^{-x}+5)^3} + C$$

Score: /3

c. $\displaystyle\int_0^{2/5} \frac{du}{\sqrt{4-25u^2}}$

If $u = 2x/5$, then $\frac{du}{dx} = 2/5$, so $du = \frac{2}{5}\,dx$. Thus

$$\int_0^{2/5} \frac{du}{\sqrt{4-25u^2}} = \int_{u=0}^{u=2/5} \frac{1}{\sqrt{4-25(\frac{2}{5}x)^2}}\frac{2}{5}\,dx = \frac{2}{5}\int_0^1 \frac{1}{\sqrt{4-4x^2}}\,dx$$
$$= \frac{2}{5}\int_0^1 \frac{1}{2\sqrt{1-x^2}}\,dx = \frac{1}{5}\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= \frac{1}{5}\sin^{-1}(x)\Big|_0^1 = \frac{\pi}{10}$$

Score: /4

Problem 5: Use the Fundamental Theorem of Calculus to evaluate the following.

a. Let $f(x) = \displaystyle\int_{x^2}^{5/x} \frac{1+t^2}{\sin(t)}\,dt$. Find df/dx.

If $g(x) = \int_0^x \frac{1+t^2}{\sin(t)}\,dt$, then $g'(x) = \frac{1+x^2}{\sin(x)}$ (by the FTC) and $f(x) = g(5/x) - g(x^2)$. Therefore $f'(x) = -g'(5/x)\frac{5}{x^2} - 2xg'(x^2) = -\frac{1+(5/x)^2}{\sin(5/x)}\frac{5}{x^2} - 2x\frac{1+x^4}{\sin(x^2)}$

b. Find $\displaystyle\frac{d}{dx}\left(\int_2^{15} ue^{3u^4-2}\,du\right)$.

Since $\int_2^{15} ue^{3u^4-2}\,du$ is a constant, its derivative is zero.

Score: /3

/13

Test 1

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Calculators permitted from here on.

Problem 6: Speedy Gonzales moves back and forth in a straight tunnel, attracted to bits of cheddar cheese alternately introduced to and removed from the ends (right and left) of the tunnel. His velocity (in m/s) is described by

$$v(t) = 0.2 + \frac{3\sin(\pi t/4)}{t}$$

Estimate change in displacement in the first 10 seconds from Speedy's original position at $t = 0$. Use rectangular approximation method to fill the table below accurate to 4 decimal places. Specify your graphing calculator commands above the table. Find the limit next to the table.

$Y_1 = 0.2 + 3\sin(\pi x/4)/x$ Use RAM with $A = 0.000000001$ and $B = 10$.
$\text{fnInt}(Y_1, X, 0, 10) \approx 6.6675$

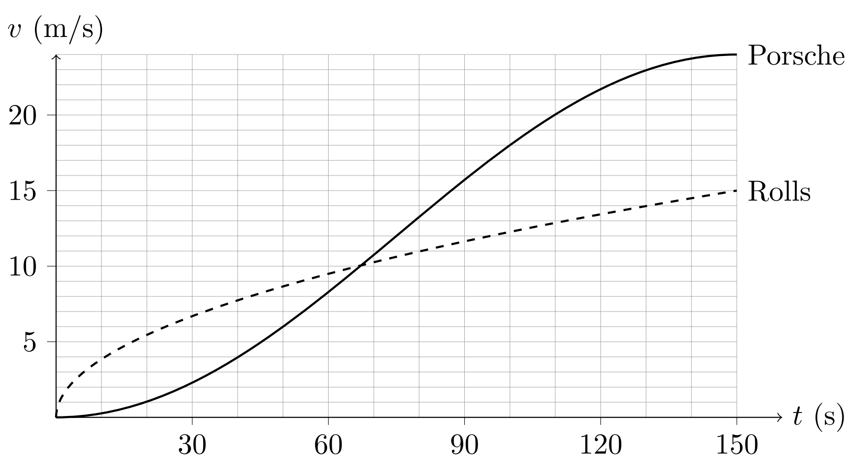
Approx	$n = 10$	$n = 100$
L_n	7.6930	6.7703
M_n	6.6688	6.6675
R_n	5.6368	6.5647

Evaluate total distance travelled by Speedy in the first 10 seconds. Provide 4 decimal places.

$$\text{displacemet is } \int_0^{10} v(t)\,dt \approx 6.6675\,\text{m}; \quad \text{distance is } \int_0^{10} |v(t)|\,dt \approx 7.8740\,\text{m}$$

Score: /5

Problem 7: A Porsche and a Rolls start from rest at a traffic light and accelerate for several minutes. The figure below shows their velocities as a function of time.



a. Which car is ahead after 70 s? Provide reason.

The Rolls is ahead because the area under its curve is greater than that of the Porsche.

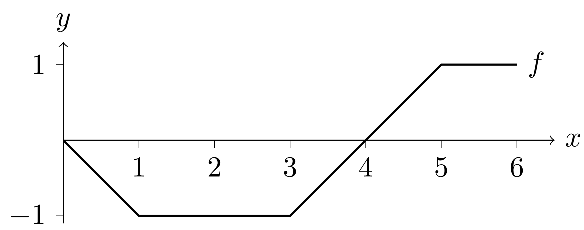
b. Which car is ahead after 2.5 minutes? By how much? Provide reason and supporting computation.

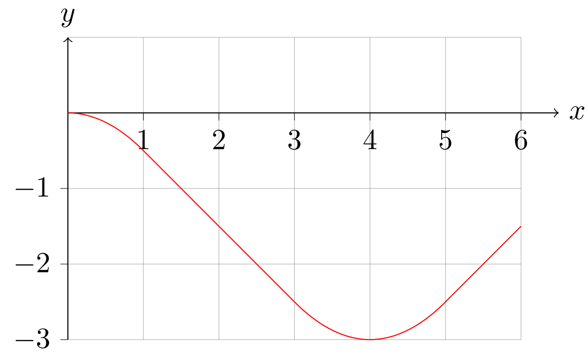
Now the Porsche is ahead. The distance lost in the first minute (actually 67 s) has been regained after about two minutes (actually 116 s), so the Porsche has the last half minute to get ahead by about 300 m.

Score: /4

/9

Problem 8: Define $A(x) = \int_0^x f(t)\,dt$ for x in $[0, 6]$, where the function f is shown below.





a. Evaluate $A(4) - A(2) = \boxed{-3/2}$

b. Within the open interval $(0, 6)$, where does A attain an extremum? An absolute minimum or an absolute maximum?

Since $A'(x) = f(x) \leq 0$ if $x < 4$ and $A'(x) = f(x) > 0$ if $x > 4$, A attains its absolute minimum at $x = 4$.

c. Order the quantities from least to greatest: (i) $A(6)$; (ii) $|A(4)|$; (iii) $A(6) - A(4)$.

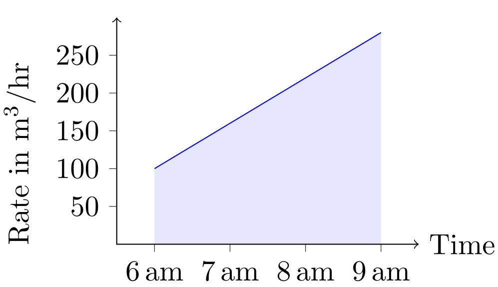
$A(6) < A(6) - A(4) < |A(4)|$

d. Graph $y = A(x)$ on the grid provided.

Score: /5

Problem 9: Burnaby wishes to measure the quantity of water that is piped to the oil refinery during a typical morning. A gauge on the water line gives the flow rate (in cubic metres per hour) at any instant. The flow rate is about $100\,\mathrm{m}^3/\mathrm{hr}$ at 6 am and increases steadily to about $280\,\mathrm{m}^3/\mathrm{hr}$ at 9 am.

a. Sketch the flow rate in the time interval given. Clearly label your axes and the graph.



b. Using only this information, give your best estimate of the total volume of water used by the refinery between 6 am and 9 am.

The shaded region above has area $\frac{100\,\mathrm{m}^3/\mathrm{hr} + 280\,\mathrm{m}^3/\mathrm{hr}}{2} \times 3\,\mathrm{hr} = 570\,\mathrm{m}^3$.

Score: /4

Problem 10: Miles jumped out of an airplane but his parachute failed to open. His downward velocity in m/s, t seconds after the jump was approximated by $v(t) = \dfrac{g}{k}(1 - e^{-kt})$, where $g = 9.8\,\mathrm{m/s}^2$ and $k = 0.2\,/\mathrm{s}$.

If he jumped from 5000 m above the ground, how long would it take Miles to hit the ground?

The distance Miles has fallen after t seconds is given by $d(t) = \int_0^t v(\theta)\,d\theta = \frac{g}{k}\int_0^t (1 - e^{-k\theta})\,d\theta = \frac{g}{k}(\theta + \frac{1}{k}e^{-k\theta})\Big|_0^t = \frac{g}{k}t + \frac{g}{k^2}e^{-kt} - \frac{g}{k^2}$. Using a graphing calculator to solve $5000 = d(t) = \frac{g}{k}t + \frac{g}{k^2}e^{-kt} - \frac{g}{k^2}$ yields that $t = 107.0\,\mathrm{s}$.

Score: /5

/14