

Math 126  
Fall 2022  
Dr. Lily Yen

Test 3  
Show all your work

Full Name: \_\_\_\_\_  
Student Number: \_\_\_\_\_  
Signature: \_\_\_\_\_  
Score: \_\_\_\_/35

**No Calculator permitted** in this part. Read the questions carefully. Show all your work and clearly indicate your final answer. Use proper notation.

**Problem 1:** Determine whether each sequence below converges as  $n \rightarrow \infty$ . If so, find the limit, else state why you cannot find the sum.

a. Write out the first 3 terms of the sequence,  $\left\{ \frac{2n - (-1)^{n-1}}{5n + (-1)^n} \right\}_{n=0}^{\infty}$ , before determining convergence as  $n \rightarrow \infty$ .

b. Write out the first 3 terms of  $s_n = \sum_{k=1}^n \frac{\sec(k\pi)}{k+1}$ , for natural numbers  $n$  before determining convergence as  $n \rightarrow \infty$ .

c. Write out the first 3 terms of  $s_n = \sum_{k=0}^n \left( \frac{5^k}{(-4)^{2k}} \right)$  before determining convergence as  $n \rightarrow \infty$ .

Score: \_\_\_\_/8

**Problem 2:** Solve for  $y$  in the following initial value problem. Express your solution explicitly as  $y = f(x)$ . Include the domain of  $f$ .

$$x \frac{dy}{dx} - e^y = 0, \quad y(1) = 2.$$

Score: /5

**Problem 3:** Find the orthogonal trajectories of the family of curves  $kx^3 = y^2$ , where  $k$  is an arbitrary constant.

Score: /4

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**Calculators permitted** in this part.

**Problem 4:** Graph the sequence  $a_n$  satisfying the recurrence:  $a_n = 3a_{n-1} - a_{n-2}$  for integers  $n \geq 2$  and  $a_0 = 1$ , and  $a_1 = 1$ . Determine  $a_5$  and whether  $a_n$  is convergent.

Score: /4

**Problem 5:** For the series

$$S = \sum_{k=1}^{\infty} \frac{(-1)^k k^2}{2^k}$$

- a. Graph the  $n$ -th partial sum of the series for the first 10 values of  $n$ . State the first three terms explicitly to support your graph.
  
  
  
  
  
  
  
  
  
  
  
- b. Determine whether the series  $S$  converges absolutely.

Score: /5

**Problem 6:** Recall Newton's law of cooling: the rate of change in temperature of an object varies proportionally to the difference in temperature between the object and the surrounding medium. Write a differential equation whose solution is the temperature as a function of time for a bottle of frozen black currant juice taken out of a  $-18^{\circ}\text{C}$  deep freezer and left in a  $25^{\circ}\text{C}$  kitchen, given that after one hour, the temperature of the bottle rose  $7^{\circ}\text{C}$ . Include all conditions. Solve the equation for the temperature of the object at time  $t$ .

Score: /4

**Problem 7:** A tank holds 100 litres of pure water. A tap is opened and a solution of 70 % alcohol and 30 % water is fed into the tank at a rate of 5 L/min. Simultaneously, a drain plug at the bottom of the tank is opened and the thoroughly mixed solution leaves the tank at the same rate. How much water is in the tank after 75 minutes? Express your answer correct to 2 decimal places.

Score: /5