Math 126
Fall 2016
Dr. Lily Yen

Test 1

Show all your work

Name: _____

Score: ___/46

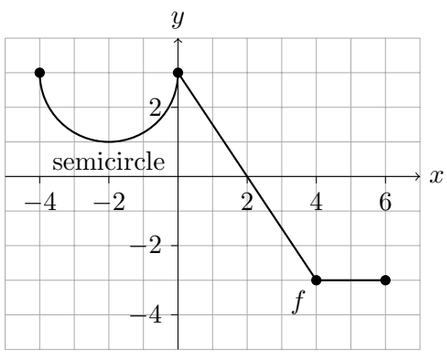
No Calculator permitted in this part. Read the questions carefully. Show all your work and clearly indicate your final answer. Use proper notation.

Problem 1: Find an explicit expression for $f(t)$ in the following definite integral.

$$\lim_{n\to\infty}\left(\sum_{k=1}^{n}\sin\left(5+\frac{(\pi-3)k}{n}\right)\frac{1}{n}\right)=\int_{5}^{\pi+2}f(t)\,dt$$

Score: /2

Problem 2: Consider the following function f defined for x in $[-4,6]$, and answer the questions below. Exact evaluations only.



a. $\displaystyle\int_{-4}^{0} f(t)\,dt$

b. $\displaystyle\int_{0}^{6} f(x)\,dx$

c. $\displaystyle\int_{0}^{6} |f(u)|\,du$

d. $\displaystyle\left|\int_{-4}^{6} f(x)\,dx\right|$

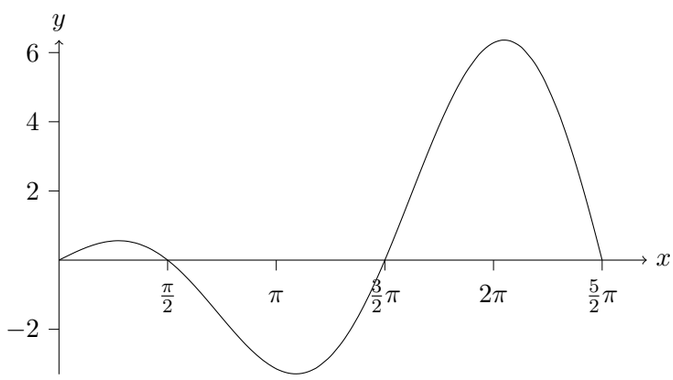
e. Directly on the grid, draw M_5 for $\int_{-4}^{6} f(x)\,dx$.

Score: /7

Problem 3: Find the expression of a Riemann sum with n intervals using the rectangular approximation method on the midpoints to estimate the area bounded by $f(x) = 3x - x^2$ and the x-axis. Draw the region of interest and M_n for a particular n.

Score: /4

Problem 4: The graph below shows $f(x) = x\cos(x)$. Let $F(x) = \int_0^x t\cos(t)\,dt$.



a. Locate the x-value(s) in $[0, 5\pi/2]$ where $F(x)$ attains local maxima.

b. Determine the x-value(s) in $[0, 5\pi/2]$ where $F(x)$ attains the global maximum.

c. How many zeroes (roots) does $F(x)$ have in $[0, 5\pi/2]$?

d. Label the inflection points of $F(x)$ with A, B, etc. on $[0, 5\pi/2]$ directly on the graph of f above. For each one, state whether the concavity changes from up to down or from down to up.

Score: /5

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Calculators permitted from here on.

Problem 5: Integrate the following analytically.

a. $\displaystyle\int_{x/3}^{x/5} \sec^2(u)\,du$

Score: /2

b. $\displaystyle\int (t+9)^{-3}\,dt$

Score: /2

c. $\displaystyle\int \csc^5(\theta)\cos(\theta)\,d\theta$

Score: /3

d. $\displaystyle\int \frac{1}{\sqrt{16-9x^2}}\,dx$

Score: /3

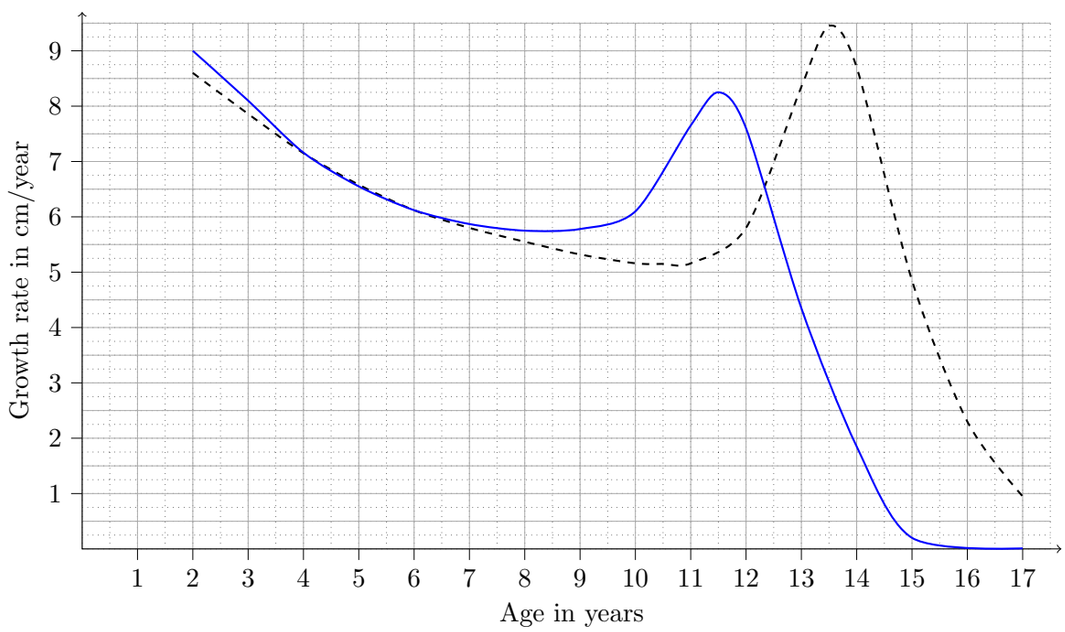
e. $\displaystyle\int \cot(x)\ln(\sin(x))\,dx$

Score: /4

Problem 6: Lily drives a distance of 20 km, accelerating uniformly from rest to 60 km/h. Graph her velocity versus time. How long does it take for her to reach her final speed?

Score: /3

Problem 7: The graphs below show the height velocity curves of Alexandra and Bartholomew between ages 2 and 17. Assuming that Alexandra and Bartholomew attain average heights in Canada, answer the following questions.



a. Which curve represents Alexandra's? Explain how you can tell.

b. Compare the start of Alexandra's growth spurt to the start of Bartholomew's. Give their ages before you compare them.

c. At the highest rate of growth, how many centimetres a year does Bartholomew grow?

Score: /3

Problem 8: The traffic flow rate past a certain point on a highway is

$$q(t) = 3000 + 2000t - 300t^2$$

(t in hours), where $t = 0$ is 8 AM. Use rectangular approximation method to estimate the number of cars passing by from 8 AM to 10 AM by filling the table below.

Approx	$n = 10$	$n = 100$	$n = 200$
L_n			
R_n			

What is the limit? Explicitly write out functions/program(s) you use or enter in your graphing calculator.

Score: /4

Problem 9: Evaluate the following exactly using the Fundamental Theorem of Calculus.

a. Let $f(\theta) = \displaystyle\int_1^{2\theta} \cot(u)\, du$. Find $f'(\theta)$.

Score: /1

b. Find $\dfrac{d}{dx}\displaystyle\int_{\sin(x)}^{\ln(x)} \frac{t}{1+t}\, dt$.

Score: /2

c. $\dfrac{d}{dx}\displaystyle\int_e^{4\pi} \log(t^2 + 3t)\, dt$

Score: /1