Math 124
Spring 2008
Dr. Lily Yen

Final Exam

Show all your work

Name: ______________

Number: ______________

Signature: ______________

Score: ___/100

- No book.
- No notes.
- Show all relevant work.
- Anything irrelevant that is wrong will be counted against you.

Problem 1: The following questions are one mark each. Provide short answers for each.

a. State the converse of $p \to q$.

b. Negate the proposition $\forall x \big(p(x) \to q(x)\big)$.

c. Evaluate $P(8, 2)$.

d. How many edges are in the complete bipartite graph $K_{3,5}$?

e. In how many ways can you arrange the letters in the word TRINOMIAL?

f. How many different ways can 10 questions on a true–false test be answered?

g. In how many ways can you arrange 10 keys on a key ring?

h. If 12 points lie on the circumference of a circle, how many inscribed pentagons can you draw having these points as vertices?

i. In how many ways can 40 things be divided into 5 equal groups?

j. A deck of cards has thirteen cards of each suit. How many 11-card hands exist that have exactly 3 aces and 4 queens?

k. Simplify the sum $\sum_{k=0}^{27} \binom{27}{k} =$ $\boxed{}$

l. Convert the binary number 11001 to decimal.

m. State an inverse for 4 modulo 7.

n. How many integer triples (i, j, k) exist that satisfy the inequality $1 \leq i \leq j \leq k \leq 50$?

o. Find the coefficient of x^2y^3 in $(2x - 3y)^5$.

p. How many spanning subgraphs does $G = (V, E)$ have if $|V| = n$ and $|E| = 5$.

Score: /16

Problem 2: Suppose $p(x, y)$ and $q(x, y)$ are propositional functions over the real numbers:

$$p(x, y) : x^2 < y + 1$$
$$q(x, y) : x^2 + y^2 = 9$$

For each of the following propositions, state whether they are true (T), false (F), or cannot be determined (?). If true, explain. If false, give a counterexample. If it cannot be determined, say why. One mark is for the right answer, and one mark for your explanation.

a. $p(-4, 10) \wedge q(0, -3)$ True or False or ?

b. $\neg p(2, 7) \vee q(-1, -2)$ True or False or ?

c. $\forall x \exists y [p(x, y)]$ True or False or ?

d. $\exists x \forall y [q(x, y)]$ True or False or ?

e. $\exists x \exists y [p(x, y) \rightarrow q(x, y)]$ True or False or ?

Score: /10

Problem 3: Consider the statement: "The square of an odd integer is an odd integer."

- Restate the statement in logical symbols using "if–then." Score: /2

- Write the negation of the statement symbolically. Score: /2

- Prove the correct statement. (Declare which one you are going to prove.) Score: /4

Problem 4: Consider the following Pascal program segment, where i, and j are integer variables.

```
For i := 1 to n do
   For j :=1 to i do
       Writeln (i*j);
```

a. If n is 10, how many times is the statement Writeln executed? Score: /2

b. Find a Θ notation in terms of n for the number of times the statement Writeln is executed. Score: /2

Problem 5: Let $A = \{a, b, c\}$ and $B = \{0, 1\}$.

a. Find the Cartesian product of A and B, i.e. $A \times B$. Score: /2

b. Find the cardinality of the power set of $A \cup B$, i.e. $|\mathcal{P}(A \cup B)|$. Score: /1

Problem 6: Let the universe U consist of all finite length binary strings. For $i \geq 1$, define A_i to be the set of all binary strings with exactly i zeroes and fewer than i ones.

a. List the elements of the set A_2. Score: /1

b. Give the set : $A_4 \cap A_9$. Score: /1

c. Describe the set $\overline{A_3}$. Score: /1

d. Describe the set: $\bigcup_{i \in \mathcal{N}} A_i$ Score: /2

Problem 7: Let $g = \{(1,a),(2,c),(3,c)\}$ be a function from $X = \{1,2,3\}$ to $Y = \{a,b,c,d\}$.

a. Is g one-to-one?

b. Is g onto Y?

c. Let $T = \{1,3\}$. Find $g(T)$

d. Let $U = \{c\}$. Find $g^{-1}(U)$.

Score: /4

Problem 8: Use the Binomial Theorem to expand $(a - 3b)^5$ completely. Simplify all coefficients and write them as integers.

Score: /4

Problem 9:

a. Use the Euclidean algorithm to find the $\gcd(1365, 550)$. Score: /2

b. Use your work above to find integers a and b such that $1365a + 550b = \gcd(1365, 550)$.
Score: /3

Problem 10: Perform the following addition in hexadecimal number system.

$$A2C + D5F7$$

Express your answer in both hexadecimal and decimal numbers. Score: /4

Problem 11: Use Mathematical Induction to prove the following proposition for all integers $n \geq 1$:

$$1 + 3 + 5 + \cdots + (2n - 1) = n^2$$

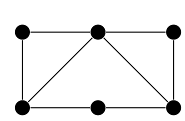
Score: /5

Problem 12: Use the Pigeonhole Principle to prove that given any set of 52 distinct integers, there must be two integers in the set whose sum or difference is divisible by 100. A complete answer includes a description of the objects, the boxes, the rule for placing objects in the boxes and an explanation of your answer. Score: /5

Problem 13: For each of the following, draw a picture of a simple graph with the desired property or explain why no such graph exists.

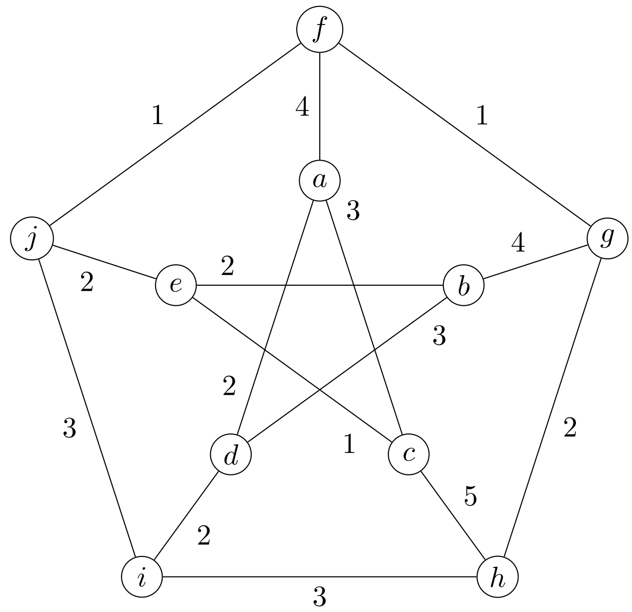
a. A connected graph with one cycle and more vertices than edges. Score: /2

b. A graph with vertex degrees 4, 3, 3, 2, 2, and 2 that is not isomorphic to the graph shown.



Score: /2

Problem 14: Below is a weighted Petersen Graph.

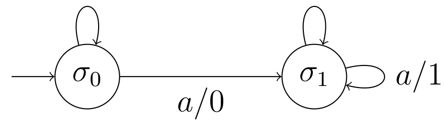


a. Find a minimal spanning tree. Score: /3

b. Does the graph have an Euler cycle? Explain. Score: /1

c. Is the Petersen Graph bipartite? Score: /1

Problem 15: Describe the language accepted by the finite-state machine given below.



Score: /3

Problem 16: Draw the transition diagram of a finite-state automaton that accepts an even number of b's over all strings over $\{a, b\}$. Score: /4

Problem 17: Give four different yet equivalent ways of characterizing a tree. Choose two of the four to prove that they are equivalent. Score: /5

Problem 18: Prove that if G is a connected graph and every vertex has even degree, then G has an Euler cycle. For partial credits, define an Euler cycle. Score: /6