Midterm One

Show all your work

Problem 1: Convert each of the following into Hindu-Arabic numeral base-10.

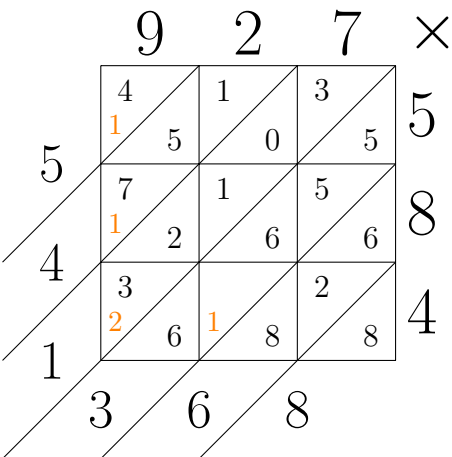   a. 10101011_2

   b. ⊓ ⋘⊓ ⋘⊓⊓

   c. V ⋀ W

   a. $2^7 + 2^5 + 2^3 + 2^1 + 2^0 = 128 + 32 + 8 + 2 + 1 = 171.$

   b. $2 \times 60^2 + 32 \times 60 + 23 = 9143.$

   c. $7 \times 20^2 + 3 \times 20 + 9 = 2869.$

Score: /5

Problem 2: Multiply 927×584 using the galley method.

Score: /2

Problem 3: We know that we have between 150 and 200 books in our STEM library. If we organize the books in groups of 8s, there are 3 left over; in groups of 6s, 5 books left over. How many books do we have?

Say you have a groups of 8 or b groups of 6. Then total number of books is $8a + 3 = 6b + 5$. For example, $8 \times 1 + 3 = 11 = 6 \times 1 + 5$.

$a =$	1	4	7	10	13	16	**19**	**22**	25
$b =$	1	5	9	13	17	21	**25**	**29**	33
Total $8a + 3 = 6b + 5 =$	11	35	59	83	107	131	**155**	**179**	203

So the STEM library has either 155 or 179 books.

Score: /3

/10

Problem 4: Sarah goes on a trip and packs 6 different tops, 3 different skirts, and 5 different pairs of shoes. How many different outfits can Sarah wear on this trip by choosing one top, one skirt, and one pair of shoes?

$$\boxed{90}$$;

$6 \times 3 \times 5 = 90$ outfits.

Score: /2

Problem 5: Yuja has 15 coins in her wallet which are either nickels (5-cent coins) or quarters (25-cent coins). She has a total of \$2.35 worth of coins in her wallet. How many nickels does Yuja have?

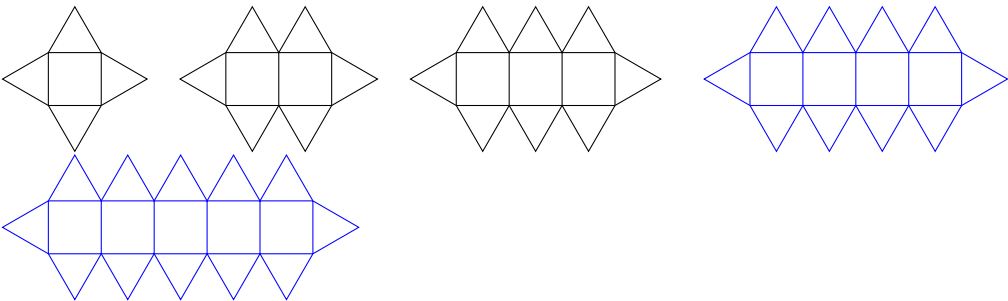Say she has x nickels and $15 - x$ quarters. Then the value in cents of her coins is

$$5x + 25(15 - x) = 235,$$

so $5x + 375 - 25x = 235$, so $-20x = 235 - 375 = -140$, so $x = 7$.
Therefore Yuja has **seven nickels** and eight quarters.

Score: /3

Problem 6: Using matchsticks, we create shapes in the following pattern. From the left, we see the first, the second, and the third.

a. Draw the 4th and the 5th shapes.

b. Draw a table of values showing a pattern for the number of matchsticks used in the first four shapes?

Shape #	1	2	3	4	5	...	n
Matchsticks	12	19	26	33	40	...	$7n + 5$

c. How many matchsticks are in the nth shape? Express your formula in terms of n.

The nth shape has $2 + 2n$ triangles that contribute $3(2 + 2n)$ matchsticks. In addition, we find $n - 1$ vertical sticks. The total is therefore

$$3(2 + 2n) + n - 1 = 7n + 5.$$

Score: /5

$$\boxed{/10}$$

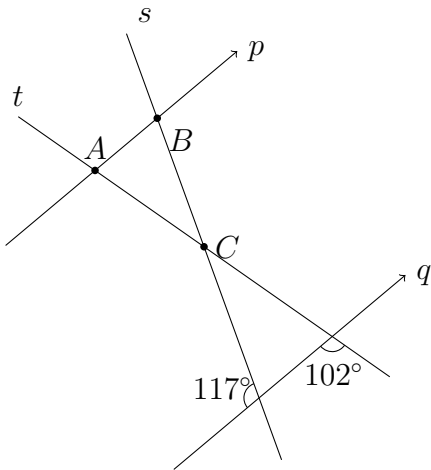**Problem 7**: Lines p and q are parallel. Lines s and t are transversals. Find the measures of INTERIOR ANGLES in $\triangle ABC$: $\angle BAC = \boxed{78^\circ}$, $\angle ABC = \boxed{63^\circ}$, and $\angle ACB = \boxed{39^\circ}$.

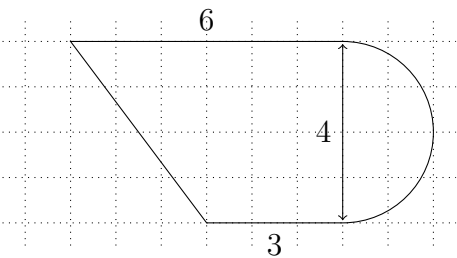<div align="right">Score: /3</div>

Problem 8: We have a slice of a circular pie with a diameter of $8\,\text{cm}$. The area of this slice is $32\,\text{cm}^2$. What is the perimeter of this slice?
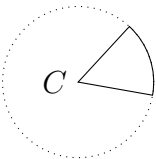
Since the diameter is $8\,\text{cm}$, the radius is $r = 4\,\text{cm}$. The area of the whole circle is $\pi r^2 = 16\pi\,\text{cm}^2$, so the sector is $\frac{32}{16\pi} = \frac{2}{\pi}$ of the circle. The circumference of the whole circle is $2\pi r = 8\pi\,\text{cm}$, so the length of the sector arc is $\frac{2}{\pi} \times 8\pi\,\text{cm} = 16\,\text{cm}$.
Therefore the perimeter of the sector is $2r + 16\,\text{cm} = 24\,\text{cm}$.

<div align="right">Score: /3</div>

Problem 9: We have a lot in the following shape consisting of a trapezoid with bases of lengths 3 and 6 that has a semicircle of diameter 4 attached to its side. Find its area and perimeter.

a. For area:

The radius of the semicircle is 2, so the total area is

$$\frac{3+6}{2} \times 4 + \frac{1}{2} \times \pi 2^2 = 18 + 2\pi \approx 24.28$$

b. For perimeter:

The slanted line has length $\sqrt{3^2 + 4^2} = \sqrt{25} = 5$, so the perimeter is

$$3 + 5 + 6 + \frac{1}{2} \times 2\pi 2 = 14 + 2\pi \approx 20.28$$

<div align="right">Score: /4</div>

<div align="right">$\boxed{/10}$</div>