Math 123-02
Summer 2024
Dr. Lily Yen

Midterm 1

Show all your work

Name: ______________
Number: ______________
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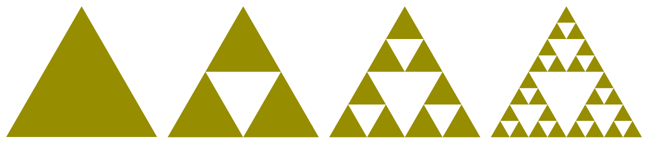
Score: ___/40

Problem 1: Sierpinski triangle is a fractal: first take an equilateral triangle (stage 0), then remove the middle triangle (stage 1), and so on by continuing to remove middle triangles from each shaded ones. Below shows stages 1, 2, and 3 following stage 0.



Tabulate the number of *shaded* triangles at each stage for at least stages 0, 1, 2 and 3, and conclude with a formula giving the number of shaded triangles at the n-th stage.

Stage	0	1	2	3	…	n
shaded triangles	1	3	9	27		3^n

Score: /4

Problem 2: Fire Horse would like to choose cellphone numbers for her family. Suppose all the numbers must begin with 778 followed by 7 digits, and their family's favourite digits are $\{3, 5, 7, 8, 9\}$, how many choices are there if Fire Horse only uses her family's favourite digits, not necessarily all five? Drawing a correct tree diagram may help.

$5^7 = 781\,125$.

Score: /3

Problem 3: Anjali's dormitory is surrounded by mature trees with many birds and squirrels. One morning Anjali noticed that there were 3 more birds than squirrels. However, when one more bird showed up, the number of birds was twice the number of squirrels. How many squirrels were there?

Let x be the number of squirrels in the beginning. Then there were $x + 3$ birds in the beginning.
Later there were $x + 4$ birds and (still) x squirrels. So $x + 4 = 2x$, so $4 = x$.
There were four squirrels.

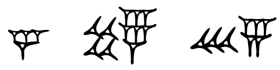
Score: /3

/10

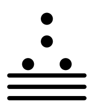
Problem 4: Translate each of the following into Hindu-Arabic numerals.

a. MCMLXXIV

$1000 + (1000 - 100) + 50 + 10 + 10 + (5 - 1) = 1974$

b. 

$4 \times 60^2 + 47 \times 60 + 35 \times 60^0 = 17\,255$

c. 

$1 \times 18 \times 20 + 1 \times 20 + 17 \times 1 = 397$

d. 324_8

$3 \times 8^2 + 2 \times 8 + 4 \times 8^0 = 212$

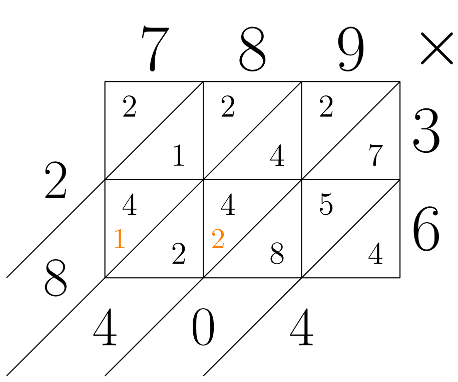
Score: /11

Problem 5: Write 5204_{10} in Kaktovik numerals.

$5204 = 13 \times 20^2 + 0 \times 20 + 4 \times 20^0 =$ 

Score: /3

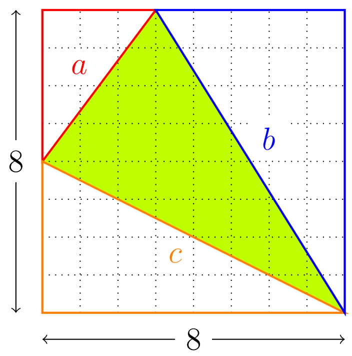
Problem 6: Use the Galley Method to perform the following: 36×789



Score: /3

/17

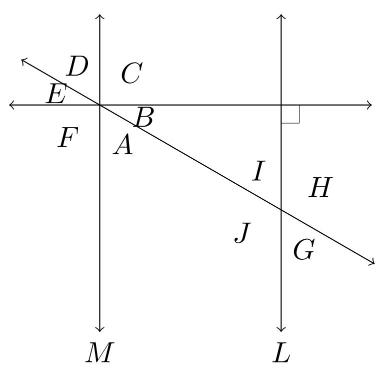
Problem 7: Find the perimeter and area of the shaded triangle in the middle of the grid shown.



Name the sides of the triangle as shown. Then $a^2 = 4^2 + 3^2 = 25$, so $a = 5$;
$b^2 = 5^2 + 8^2 = 89$, so $b = \sqrt{89}$; and $c^2 = 4^2 + 8^2 = 80$, so $c = \sqrt{80} = 4\sqrt{5}$. Therefore the perimeter is $a + b + c = 5 + \sqrt{89} + 4\sqrt{5} \approx 23.378$.
The area of the whole square is $8^2 = 64$. The areas of the three triangles in the corners are $\frac{3\times4}{2} = 6$, $\frac{5\times8}{2} = 20$, and $\frac{4\times8}{2} = 16$. The area of the middle triangle is therefore $64 - (6 + 20 + 16) = 22$ square units.

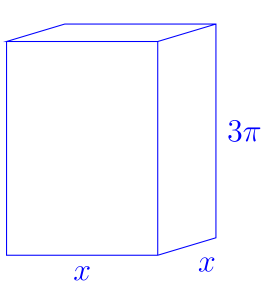
Score: /5

Problem 8: Given $\angle D = 58°$, find the measures of $\angle A = \boxed{58°}$, $\angle B = \boxed{32°}$, $\angle I = \boxed{58°}$, and $\angle J = \boxed{122°}$.

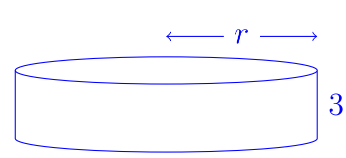


Score: /4

Problem 9: Draw a square-based rectangular prism of height 3π cm and a volume of $12\pi\,\text{cm}^3$. Suppose that a right cylinder of height 3 cm also have the same volume as the rectangular prism. Which solid has a bigger surface area? Show all steps.



The volume is $3\pi x^2 = 12\pi$, so $x^2 = 4$, so $x = 2\,\text{cm}$. The surface area is therefore $2x^2 + 4x \times 3\pi = 8 + 24\pi \approx 83.40\,\text{cm}^2$.



The volume is $\pi r^2 \times 3 = 12\pi$, so $r^2 = 4$, so $r = 2$. The surface area is therefore $2\pi r^2 + 2\pi r \times 3 = 8\pi + 12\pi = 20\pi \approx 62.83$ square cm.
The box has the greater surface area.

Score: /4

/13