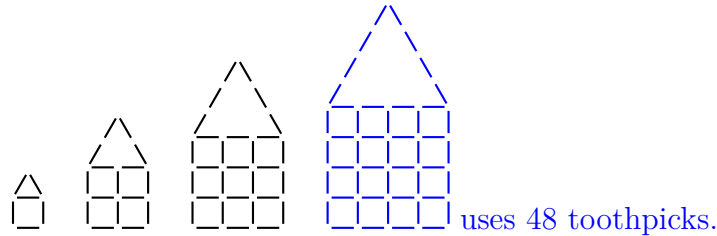


Midterm

Show all your work

Name: _____
 Number: _____
 Signature: _____
 Score: ____/35

Problem 1: Anjali used toothpicks to form a sequence of houses.



- Draw the fourth house in the sequence.
- Count the total number of toothpicks in the fourth picture.
- Establish a formula $H(n)$ for the total number of toothpicks in the n th house.

House	1	2	3	4	...	n
Toothpicks	6	16	30	48	...	$2n(n+1) + 2n$

Score: /3

Problem 2: Katharina’s parents invited 4 couples and 6 singles to their home. Suppose the couples (5 in total) and 6 singles all shook hands with one another except the couples did not shake hands with their own partners, how many handshakes were there?

There were a total of $2 \times 5 + 6 = 16$ people. Without any restriction, there would be $16 \times 15/2 = 120$ handshakes. Without five couples handshakes, there would be $120 - 5 = 115$ handshakes.

Score: /2

Problem 3: Mei has a mason bee house in her garden while Mei’s brother collects spiders. If the number of bees is three less than twice the number of spiders, and the total number of bees and spiders is 12, find the total number of legs assuming that all bees and spiders have all the legs each is supposed to have: six each for bees and eight each for spiders. The following table may help you organize your work for the number of legs.

Say the number of spiders is x . Then the number of bees is $2x - 3$, so the number of bugs is $x + 2x - 3 = 3x - 3$, so $3x - 3 = 12$, so $3x = 15$, so $x = 5$. That is, 5 spiders and 7 bees, so $5 \times 8 + 7 \times 6 = 82$ legs.

Score: /2


Problem 4: Mei flips a nickel, a dime, a quarter, a loonie, and a toonie (a two-dollar Canadian coin). How many different results can she expect from flipping these five different coins? Use a tree diagram to support your answer.


Since each coin could show a head or a tail, 2 choices, Mei can expect $2^5 = 32$ outcomes.

Score: /2

Problem 5: Fire Horse is looking at the following from different numeration systems. Sort them from smallest to largest.

Each correct conversion to a Hindu-Arabic numeral earns one point.

a. 

b. 

c. MCMLXXIV

d. 1010011_2

e. $8CB_{16}$

f. 

a. $14 \times 60^2 + 26 \times 60 + 43 = 52\,003.$

b. $11 \times 20^2 + 17 \times 20 + 9 = 4749.$

c. $1000 + 900 + 50 + 20 + 4 = 1974.$

d. $64 + 16 + 2 + 1 = 83.$

e. $8 \times 256 + 12 \times 16 + 11 = 2251$

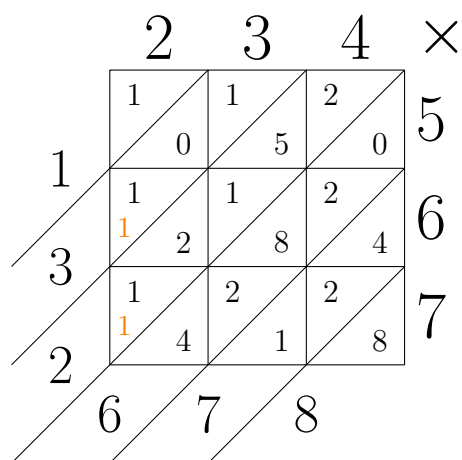
f. $1 \times 18 \times 20 + 4 \times 20 + 7 = 447.$

$d < f < c < e < b < a$

Score: /7

132 678

Problem 6: Multiply 234×567 using the galley method.



Score: /2

Problem 7: Fire Horse has a bag of chickpeas. When she plants them 5 in a row or 7 in a row, she has 3 left over in each case. If she plants them 9 in a row, she has 1 left over. Find

73

the smallest number of chickpeas that Fire Horse may have.

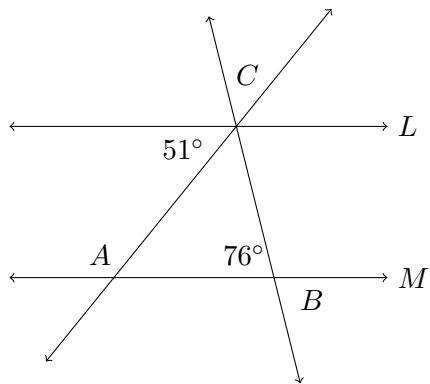
If we put 3 chickpeas aside, the remainder is a multiple of both 5 and 7, so a multiple of $\text{lcm}(5, 7) = 35$. Therefore the total number of chickpeas is one of 3, 38, 73, 108, 143, 178, 213, 248, ...

The number of chickpeas is also 1 more than a multiple of 9, so 1, 10, 19, 28, 37, 46, 55, 64, 73, 82, ...

The first number common between the two lists is 73 and the next is $73 + \text{lcm}(5, 7, 9) = 73 + 315 = 388$.

Score: /3





Problem 8: Lines L and M are parallel. Find the measures of $\angle A = \boxed{129^\circ}$,
 $\angle B = \boxed{76^\circ}$, and $\angle C = \boxed{53^\circ}$.



The given angle 51° and $\angle A$ are interior angles on the same side of the transversal, so they add to 180° . The given angle 76° and $\angle B$ are vertically opposite angles, so they have the same measure. The given angle 51° , the corresponding angle to 76° , and $\angle C$ form a straight line, so they add up to 180° .

Score: /3

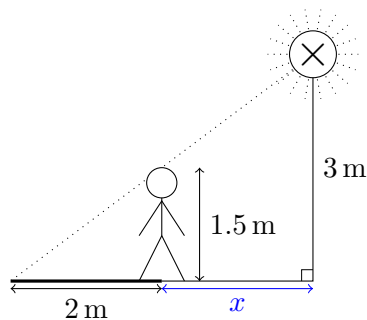
Problem 9: Set up a table for convex polygons' angle sums beginning with a triangle, followed by a quadrilateral, a pentagon, and so on. State the angle sum of an eleven-sided polygon.

Polygon:				...	
Angle sum:	180	360	540	...	1620

The angle sum of an n -gon is $180(n - 2)$.

Score: /2

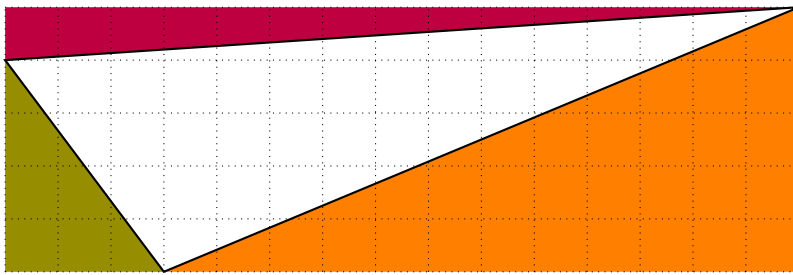
Problem 10: Shown is a 1.5 m tall pedestrian standing near a lamp post of 3 m high. If the pedestrian's shadow is 2 m long, how far away is the pedestrian standing away from the base of the lamp post?



If the distance from the pedestrian to the lamp post is x , then, by similar triangles, $\frac{x+2}{2} = \frac{3}{1.5}$, so $x + 2 = 2 \times \frac{3}{1.5} = 4$, so $x = 2$ m.

Score: /2

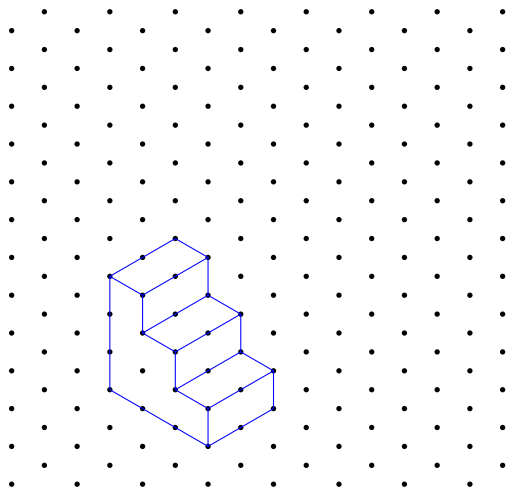
Problem 11: Below is a 5×15 grid containing a big triangle. Find the area of the big triangle. Show your work.



Use the area of the rectangle minus the area of the three corner triangles:
 $5 \times 15 - (1 \times 15 + 3 \times 4 + 12 \times 5)/2 = 75 - 43.5 = 31.5$ square units.

Score: /2

Problem 12: In the three dimensional guide below, draw a three-step staircase where each step is 2 units long by 1 unit wide, and each step rises 1 unit tall. Find the volume and surface area of the staircase.



We drew the staircase in class. See your lecture notes.

$$V = 2 \times 1 \times 1 + 2 \times 1 \times 2 + 2 \times 1 \times 3$$

$$= 12$$

$$SA = 2 \times (3 \times 3 + 2 \times 3 + 2 \times 3)$$

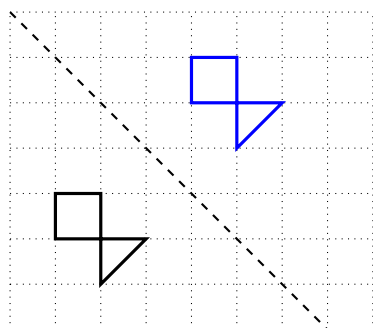
$$- 2 \times (2 \times 1 + 1 \times 1)$$

$$= 36$$

So, $V = 12 \text{ u}^3$ and $SA = 36 \text{ u}^2$.

Score: /3

Problem 13: Reflect the given figure along the given diagonal.



Score: /2