

Math 123  
Spring 2023  
Dr. Lily Yen

# Quiz 1

Show all your work

Name: \_\_\_\_\_  
Number: \_\_\_\_\_  
Signature: \_\_\_\_\_  
Score: \_\_\_\_/8

**Problem 1:** Katharina's parents love couples potluck gatherings. If her parents join 10 other couples in a potluck and they all shake hands with each other (including their own partners), how many handshakes are there? How many handshakes will there be if they don't shake hands with their own partners?

Clearly show your reasoning in a logical and organized fashion.

There are 11 couples, or 22 people, a total of  $22 \times 21/2 = 231$  handshakes without restriction. Eleven couples not shaking hands with their partners mean 11 handshakes fewer, so 220 in total

Score: \_\_\_\_/2

**Problem 2:** Below is a  $3 \times 3$  grid. Make a magic square by using all whole numbers from 1 to 9 exactly once to fill the square such that the row sum, column sum, and diagonal sum are all equal. Hint: Place 5 in the middle square and an even number at a corner.

2	9	4
7	5	3
6	1	8

We find the sum of the first 9 consecutive positive integers is 45, so  $45 \div 3 = 15$ , the row sum, column sum, and diagonal sum. The number 5 is the middle number, so should be placed in the middle of the square. The middle column, middle row, and two diagonal sums need to use all four pairs of remaining 8 integers adding to 10 for each pair. When 1 and 9 are placed in one of the diagonals, it is impossible to find four remaining integers such that 1 and two other integers could add to 15 for both the row and the column sums involving 1. This forces 1 and 9 to be in a middle column or a middle row. Once this is done, the other digits can find their places.

Score: \_\_\_\_/2

**Problem 3:** Mei, Katharina, and Anjali wanted to share a long fruit leather of one metre. If Anjali had twice as much as Mei, and Katharina had 8 centimetres less than double the combined length of Anjali and Mei's fruit leather pieces, how many centimetres did each get?

Let Mei have  $m$  centimetres out of 100 centimetres, then Anjali would have  $2m$  centimetres, and Katharina would have  $2 \times (m + 2m) - 8$ . Since the total is  $m + 2m + 2(m + 2m) - 8 = 100$ , we get  $3m + 6m - 8 = 100$ , so  $9m = 108$ , or  $m = 12$ . Mei 12 cm, Anjali 24 cm, and Katharina 64 cm

Score: /2

**Problem 4:** Anjali worked in SPCA as a volunteer. She was responsible in arranging a row of cages: 3 cages with a dog each, 2 cages with a cat each, and 1 cage with a hamster.

- a. How many ways could she arrange these 6 cages?
- b. Suppose the hamster cage must be at the end of the row, how many ways could she arrange the cages?

Use a tree diagram to support your reasoning.

Using a tree diagram, Anjali would have 6 branches first, followed by 5, 4, 3, 2, 1, so  $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$  ways. With the hamster cage fixed at the end, there are 5 cages to arrange, so  $5! = 120$  ways.

Score: /2