Math 123-01 Fall 2025 Dr. Lily Yen

## Midterm 2 Show all your work

Name: Number: Signature: Score: /30

**Problem 1**: Answer each question to two decimal places when appropriate. Steps may help with part marks.

a. Find 31.6 % of 270.

 $0.316 \times 270 = 85.32$ 

b. Convert the fraction  $5\frac{2}{7}$  into a percent.

$$5\frac{2}{7} = 5 + \frac{2}{7} = \frac{37}{7} \approx 5.286 = 528.6\%$$

 $5\frac{2}{7}=5+\frac{2}{7}=\frac{37}{7}\approx5.286=528.6\,\%$ c. When David buys a 200-dollar (ticket price) skateboard, how much does the store charge David in taxes? Hint: Stores need to charge a 5% GST and a 7% PST.

\$24

12% of \$200 is  $0.12 \times 200 = 24$ .

d. Janette treated her mother to a Mother's Day Brunch at Queen Elizabeth's Theatre's Browns Social House. She saved \$80 for this occasion to cover a  $15\,\%$  tip and a  $12\,\%$ service tax. What was the maximum they could order to stay within her budget of

\$80?

Say the cost before tip and taxes is x. Then the tip is 0.15x, and the tax is 0.12x, so she has to pay x + 0.15x + 0.12x = 1.27x.

e. The 76sidents of a small town and the surrounding areas are divided over the proposed construction of a new waste management station in town, as shown in the table. A reporter randomly selects a person to interview from a group of residents. If the person selected lives in town, what is the probability that the person supports the new waste management station?

	Support new station	Oppose new station
Live in town	252	6369
Live in surrounding areas	7518	416

3.81

$$\frac{252}{252+6369} = \frac{252}{6621} \approx 0.0381 = 3.81 \%$$

Score:

**Problem 2**: Solve for the indicated variable.

a. Solve for t in A = P(1 + rt)

If 
$$A = P(1 + rt)$$
, then  $\frac{A}{P} = 1 + rt$ , so  $rt = \frac{A}{P} - 1$ , so  $t = \frac{\frac{A}{P} - 1}{r} = \frac{\frac{A - P}{P}}{r} = \frac{A - P}{Pr}$ 

b. Solve for x in  $(2.5)^x = 37$ 

If 
$$(2.5)^x = 37$$
, then  $\log((2.5)^x) = \log(37)$ , so  $x \log(2.5) = \log(37)$ , so  $x = \frac{\log(37)}{\log(2.5)} \approx 3.941$ 

c. Solve for r in  $A = P(1 + r/m)^n$ 

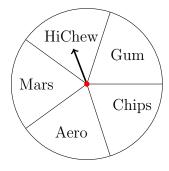
If 
$$A = P(1 + r/m)^n$$
, then  $\frac{A}{P} = (1 + r/m)^n$ , so  $(A/P)^{1/n} = 1 + r/m$ , so  $(A/P)^{1/n} - 1 = r/m$ , so

$$r = m\left(\left(\frac{A}{P}\right)^{1/n} - 1\right) = m\sqrt[n]{A/P} - m$$

Score:

/5

**Problem 3**: Dad constructed a spinner with five equal sectors the morning after Halloween for Hamlet and Samlet. Assume that the pointer never lies on a border line, answer the following questions.



- a. State the sample space for spinning the spinner once.
  - $S = \{Gum, Chips, Aero, Mars, HiChew\}$
- b. Find the probability for the event of getting at least one Aero after two spins.

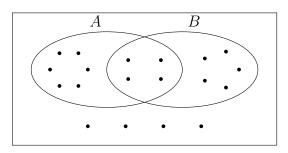
The chance of zero Aeros after two spins is  $\left(\frac{4}{5}\right)^2 = \frac{16}{25}$ , so the chance of at least one Aero is  $1 - \frac{16}{25} = \frac{9}{25} = 0.36 = 36\%$ .

c. Find the probability for the event of getting no Gum after three spins.

The chance of zero Gums is  $\left(\frac{4}{5}\right)^3 = \frac{64}{125} = 0.512 = 51.2 \%$ .

Score: /5

**Problem 4**: Dad drew a big rectangle representing a sample space containing two events, A and B. Assume that the outcomes (as dots) were all equally likely, answer the following questions.



- a.  $P(A \cup B)$

b.  $P(\bar{B})$ 

- c.  $P(B \cap \bar{A})$
- d.  $P(A \mid B)$
- e. Are A and B mutually exclusive?

 $A \cap B \neq \emptyset$ , so A and B are not mutually exclusive.

$$\frac{15}{19} \approx 78.9 \%$$

$$\frac{10}{19} \approx 52.6 \%$$

$$\frac{5}{19} \approx 26.3 \%$$

$$\frac{4}{9} = 44.4 \%$$

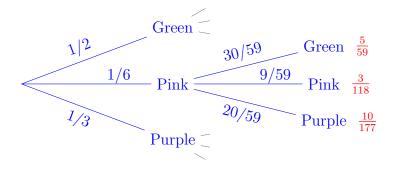
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Page 2 Math 123-01

**Problem 5**: A candy jar contains 30 green jelly beans, 10 pink jelly beans, and 20 purple jelly beans. Two jelly beans are randomly selected without replacement. Let P be the event you select a pink jelly bean first, and let N be the event the second jelly bean is not purple.

- a. Find the probability that the first jelly bean is NOT pink.
- b. Find  $P(N \mid P)$  with a probability tree.



Since  $P(\text{Pink first}) = \frac{10}{60}$ , P(Not Pink first) = 1 - 1/6 = 5/6 for part a). Moreover,

$$P(N \mid P) = \frac{P(N \text{ and } P)}{P(P)} = \frac{\frac{5}{59} + \frac{3}{118}}{\frac{1}{6}} = \frac{\frac{13}{118}}{\frac{1}{6}} = \frac{39}{59} \approx 0.661 = 66.1 \%$$

Score: /4

**Problem 6**: Suppose Brian's brother purchased a used sail boat for \$5000 and agreed to pay off the boat in 18 monthly payments of \$400 each.

a. Find the total amount of interest charged in this boat loan.

He paid a total of  $18 \times \$400 = \$7200$ , so the interest was \$7200 - \$5000 = \$2200.

b. Assume the payments were computed using the add-on interest method; find the annual interest rate applied.

With the add-on method, 
$$I = Prt$$
, so  $r = \frac{I}{Pt} = \frac{\$2200}{\$5000 \times 1.5} \approx 0.2933 = 29.33\%$ 

Score: /3

**Problem 7**: David's grandparents want to establish a fund for their grandchildren's university education. What lump sum must they deposit at a 2.5 % annual interest rate, compounded monthly, in order to have \$30 000 in the fund at the end of 18 years?

If

$$30\,000 = P\left(1 + \frac{2.5\,\%}{12}\right)^{18\times12} = P(1.002\,083)^{216},$$

then  $P = \frac{\$30\,000}{(1.002\,083)^{216}} \approx \$19\,137.80.$ 

Score: /3

/10

Page 3 Math 123-01