Math 123-01
Fall 2025
Dr. Lily Yen

Quiz One

Show all your work

Name: _____
Number: _____
Signature: _____
Score: ___/10

Problem 1: Anjali's family was invited to a friend's party. Suppose there were 4 families in total: 8 parents and 11 ($= 2 + 3 + 3 + 3$) children. If all the adults shook hands with one another except with their spouses and all the children shook hands except with their siblings, how many handshakes took place?

Adults' handshakes: $\frac{8\times 6}{2} = 24$.
Children's handshakes: $\frac{2\times 9+3\times 3\times 8}{2} = 45$.
Total $24 + 45 = 69$ handshakes.
Alternatively: All the adults could shake hands in $\frac{8\times 7}{2} = 28$ ways. But the 4 handshakes between spouses do not happen, so only $28 - 4 = 24$ handshakes between adults.
Similarly, the children could shake hands in $\frac{10\times 11}{2} = 55$ ways. Within the 2-child family, the siblings could shake hands in $\frac{2\times 1}{2} = 1$ way. That's 1 handshake that does not happen.
Similarly, with in each of the 3 families with three kids, the siblings could shake hands in $\frac{3\times 2}{2} = 3$ ways. That's $3 \times 3 = 9$ handshakes that do not happen. So there is only $55 - 1 - 9 = 45$ handshakes between children.
Again, a total of $24 + 45 = 69$ handshakes.

Score: /3

Problem 2: Katharina has a 3.6 m ribbon to cut into three pieces to wrap three presents. Suppose the longest piece is twice as long as the shortest piece, and the medium piece is the average of the longest and shortest pieces, find the length of each piece of ribbon.

Suppose the length of the shortest piece is x. The the longest has length $2x$, and the middle piece has length $\frac{x+2x}{2} = \frac{3}{2}x$.
Therefore $x + 2x + \frac{3}{2}x = 3.6\,\text{m}$, so $\frac{9}{2}x = 3.6\,\text{m}$, so $x = \frac{3.6\,\text{m}}{9/2} = 0.8\,\text{m}$. The three lengths are therefore

$$x = 0.8\,\text{m}, \quad \frac{3}{2}x = 1.2\,\text{m}, \quad \text{and} \quad 2x = 1.6\,\text{m}.$$

Score: /3

Problem 3: Calculate the following:

a)
$$\begin{array}{r} 32 \\ \times\ 28 \\ \hline 896 \end{array}$$

b)
$$\begin{array}{r} 52 \\ \times\ 48 \\ \hline 2496 \end{array}$$

c)
$$\begin{array}{r} 92 \\ \times\ 88 \\ \hline 8096 \end{array}$$

d)
$$\begin{array}{r} 102 \\ \times\ 98 \\ \hline 9996 \end{array}$$

Compare the answers with the numbers you multiplied. (Hint: Perhaps also compare with 30×30 for part a). Look for a pattern to help you determine the following two products: 8002×7998 and $1\,000\,002 \times 999\,998$.

Each of the four examples use that $(x + 2)(x - 2) = x^2 - 2^2 = x^2 - 4$. In the first example, $x = 30$, so $x + 2 = 32$, $x - 2 = 28$, and $x^2 = 900$, so $x^2 - 4 = 896$.
Similarly, if $x = 8000$, then $8002 \times 7998 = 8000^2 - 4 = 64\,000\,000 - 4 = 63\,999\,996$, and if $x = 1\,000\,000$, then
$1\,000\,002 \times 999\,998 = 1\,000\,000^2 - 4 = 1\,000\,000\,000\,000 - 4 = 999\,999\,999\,996$.

Score: /4